

## Dynamics

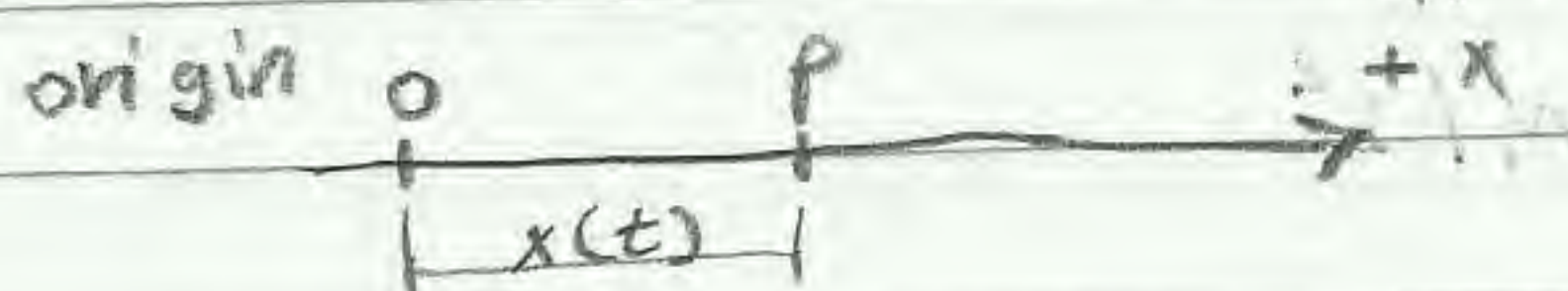
## Kinematics Kinetics

Study the properties of the motion regardless of its reasons.

relate the properties of motion & its reasons.

## Kinematics of a particle

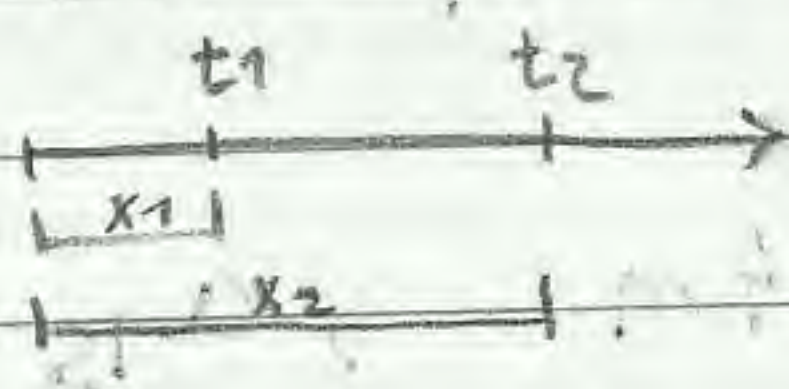
## Rectilinear motion



① position:  $x(t)$

② Displacement:  $\Delta x$

$$\Delta x_{t_1 \rightarrow t_2} = x_2 - x_1$$



③ velocity:  $v$

$$v(t) = \frac{dx}{dt}$$

④ Acceleration:  $a$

$$a(t) = \frac{dv}{dt}$$



## Example

A particle moves along a straight line such that

$$x(t) = t^2 - 2t + 5$$

calculate

- ① the initial position, velocity, acceleration.
- ② when the velocity is zero? what's the position & acceleration at this time?
- \*③ The position, displacement & total distance traveled at  $t=3s$

**Solution.**

$$① \quad x = t^2 - 2t + 5 \quad m$$

$$v = \frac{dx}{dt} = 2t - 2 \quad m/s$$

$$a = \frac{dv}{dt} = 2 \quad m/s^2$$

$$\text{at } t=0 \Rightarrow x_0 = 5m, \quad v_0 = -2m/s, \quad a_0 = 2m/s^2$$

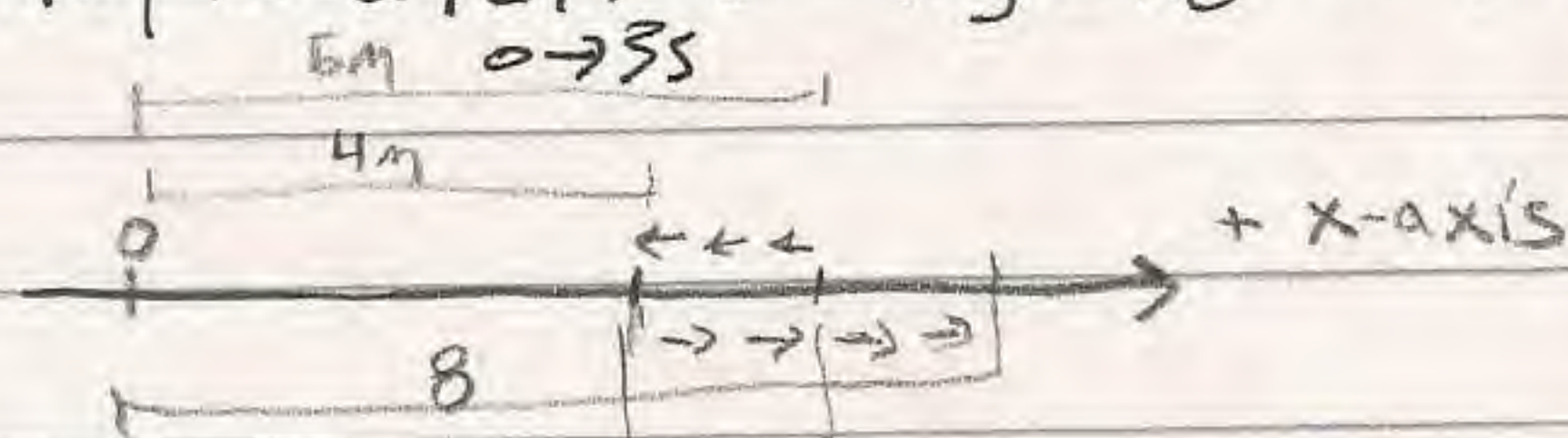
$$② \quad \text{let } v=0 \Rightarrow 2t-2=0 \Rightarrow t=1s$$

(change in direction)

$$x_{1s} = 4m, \quad a_{1s} = 2m/s^2$$

$$③ \quad \text{at } t=3s \Rightarrow x_{3s} = 3(3)^2 - 2(3) + 5 = 8m$$

$$\text{Displacement} = x_3 - x_0 = 8 - 5 = 3m$$



$t=1$	$t=0$
$x_1=4$	$x_0=5m$
$v_1=0$	$v_0=-2$
$a_1=2$	$a_0=2$

Sign of a +  $\rightarrow$  تسارع

type of a  $\left\{ \begin{array}{l} \text{accelerating rate} \quad (+,+) \text{ و } (-,-) \\ \text{decelerating rate} \quad (+,-) \text{ و } (-,+) \end{array} \right.$



total distance traveled

$$= \overset{(\leftarrow)}{1} + \overset{(\rightarrow)}{4} = 5 \text{ m}$$

total distance traveled

$$= |x_{v=0} - x_{\text{initial}}| + |x_{\text{final}} - x_{v=0}|$$

$$= |x_{15} - x_0| + |x_3 - x_{15}|$$

$$= |4 - 5| + |8 - 4| = 1 + 4 = 5 \text{ m}$$

problem 11.11 (11.11) motion

at the time when the velocity is again zero  
is position & velocity at the  
total displacement or total distance traveled at  $t = 12$

$$5 + 5 - 0 = 0$$

$$4(5 + 5 - 0) = v_0 \Rightarrow \frac{5 + 5 - 0}{10} \Rightarrow \frac{v_0}{10} \Rightarrow v_0 = 10 \text{ m/s}$$

$$10 + 5 - 0 = 15$$

$$10 + 5 - 0 = 15$$

$$10 + 5 - 0 = 15$$

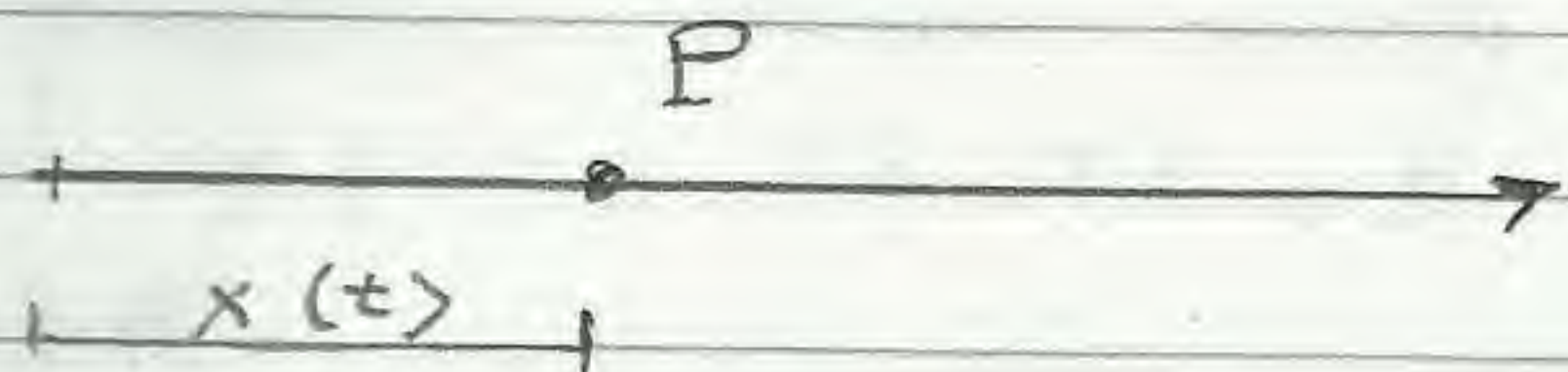
$$4(5 + 5 - 0) = v_0 \Rightarrow \frac{5 + 5 - 0}{10} \Rightarrow \frac{v_0}{10} \Rightarrow v_0 = 10 \text{ m/s}$$

$$4(5 + 5 - 0) = v_0 \Rightarrow \frac{5 + 5 - 0}{10} \Rightarrow \frac{v_0}{10} \Rightarrow v_0 = 10 \text{ m/s}$$

$$10 + 5 - 0 = 15$$



## Rectilinear motion



$$x(t) \xrightarrow[\int dt]{\frac{d}{dt}} v = \frac{dx}{dt} \xrightarrow[\int dt]{\frac{d}{dt}} a = \frac{dv}{dt}$$

$$\text{Displacement} = x_{\text{final}} - x_{\text{initial}}$$

at  $v=0 \rightarrow$  the direction must be changed.

problem (11.14) page 614

a particle in a straight line  $a = 9 - 3t^2$

at  $t=0 \rightarrow v_0=0, x_0=5\text{m}$  calculate:

- (1) The time when the velocity is again zero
- (2) position & velocity at  $t=4\text{s}$ .
- (3) The displacement & total distance travelled at  $t=4\text{s}$ .

$$a = 9 - 3t^2$$

$$\text{let } a = \frac{dv}{dt} \Rightarrow \frac{dv}{dt} = 9 - 3t^2 \Rightarrow \int dv = \int (9 - 3t^2) dt$$

$$v = 9t - t^3 + C_1$$

$$\text{at } t=0 \rightarrow v_0=0 \Rightarrow C_1=0$$

$$v = 9t - t^3$$

OR

$$\int_0^v dv = \int_0^t (9 - 3t^2) dt$$

$$|v|_0^v = |9t - t^3|_0^t \Rightarrow v - 0 = (9t - t^3) - 0$$

$$v = 9t - t^3$$



$$\text{let } v = \frac{dx}{dt}$$

$$\frac{dx}{dt} = 9t - t^3 \Rightarrow \int_5^x dx = \int_0^t (9t - t^3) dt$$

$$|X|_0^5 = \left| 4.5t^2 - 0.25t^4 \right|_0^t$$

$$X = 4.5t^2 - 0.25t^4 + 5$$

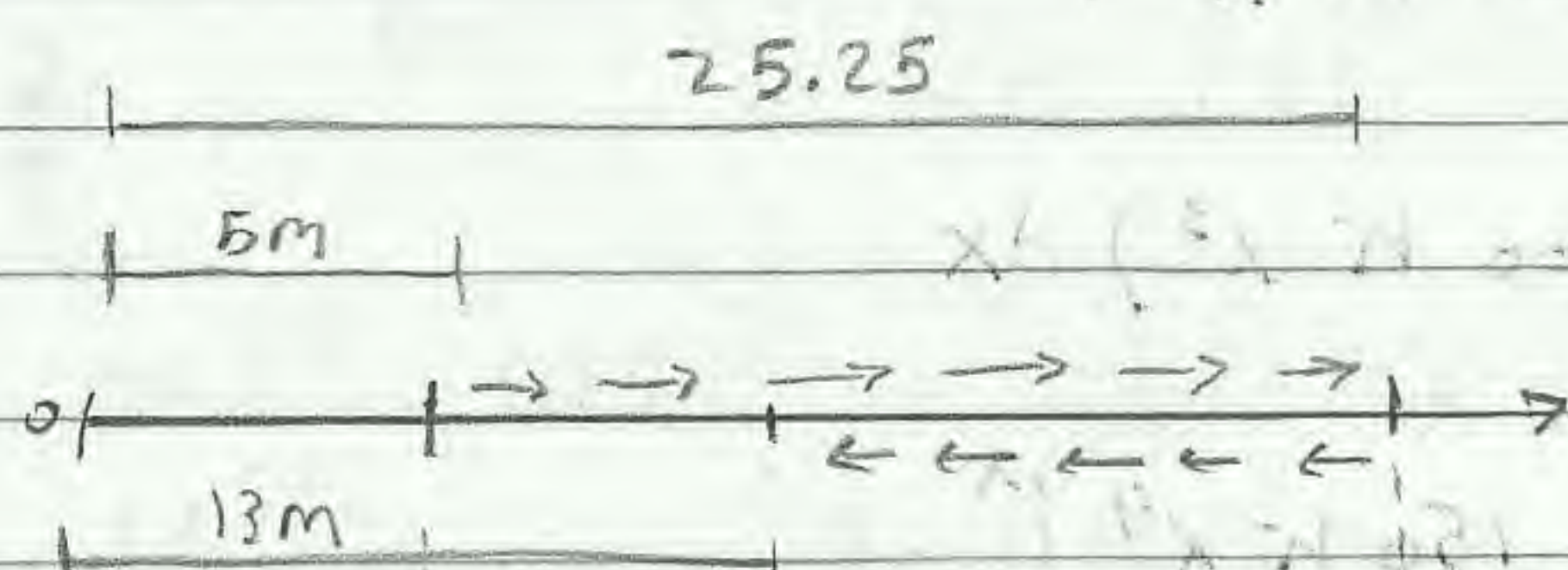
let  $V=0 \rightarrow gt - t^3 = 0 \rightarrow g - t^2 = 0 \rightarrow t = 0, t = 3, t = -3$

at  $t=0 \Rightarrow x_0=5, v_0=0, a_0=9 \text{ m/s}^2$

at  $t = 3 \Rightarrow x_3 = 25.25 \text{ m}$ ,  $v_3 = 0$ ,  $a_3 = -18 \text{ m/s}^2$

at  $t = 4 \Rightarrow x_4 = 13 \text{ m}$ ,  $v = -28 \text{ m/s}$ ,  $a_4 = -39 \text{ m/s}^2$

$$\text{Displacement} = x_4 - x_0 = 13.5 = 8 \text{ m}$$



$$\begin{aligned} t &= 4 \\ X_4 &= 13 \\ U &= -28 \\ a &= 39 \end{aligned}$$

$$t = 35$$

$$X_1 = 75.25$$

$$V_3 = 0$$

$$a_3 = -18$$

total distance =  $20.25 + 12.25 = 32.5 \text{ m}$

$$|x_3 - x_0| + |x_4 - x_3|$$

\* إذا لم يتغير  $V_0$  ، لازم اجيبها لانه ثابت Total distance



problem (11.18) page 614

$$a = 600x + 600Kx^3$$

K is a constant

$$x=0 \rightarrow v_{x=0} = 7.5 \text{ m/s}$$

$$x=0.45 \text{ m} \rightarrow v_{x=0.45} = 15 \text{ m/s}$$

calculate the velocity as a function of position  $v(x)$   
& the constant  $K$ .

$$a = \frac{dv}{dt}$$

$$a = \left( \frac{dx}{dt} \right) \left( \frac{dv}{dx} \right) = v \frac{dv}{dx}$$

$$\text{let } a = v \frac{dv}{dx}$$

$$v \frac{dv}{dx} = 600x + 600Kx^3$$

$$\int_{7.5}^v v dv = \int_0^x (600x + 600Kx^3) dx$$

$$\left| \frac{v^2}{2} \right|_{7.5}^v = \left| 300x^2 + 150Kx^4 \right|_0^x$$

$$\frac{v^2}{2} - \frac{(7.5)^2}{2} = 300x^2 + 150Kx^4$$

$$\text{at } x=0.45 \text{ m, } v=15 \Rightarrow K = \checkmark$$



problem (11.25) page 615

$$a = -k\sqrt{v}$$

$k$  is a constant.

at  $t=0 \Rightarrow x_0=0$  ( $v_0=25$  m/s)

$x=6$  m,  $v_{x=6\text{m}}=12$  m/s

calculate: \* the velocity at  $x=8$  m.

\* the time required to come to rest ( $v=0$ )

( $v-x$ ) relation.

let  $a = v \frac{dv}{dx}$

$$v \frac{dv}{dx} = -k v^{\frac{1}{2}}$$

$$\int_{25}^v \frac{v}{v^{\frac{1}{2}}} dv = -k \int_0^x dx$$

$$\frac{2}{3} |v^{\frac{3}{2}}|_{25}^v = -k |x|_0^x$$

$$v^{\frac{3}{2}} - (25)^{\frac{3}{2}} = -1.5 k x$$

$$v^{\frac{3}{2}} = 125 - 1.5 k x$$

at  $x=6$ ,  $v=12 \Rightarrow k=9.27$

$$v^{\frac{3}{2}} = 125 - 1.5(9.27)x$$

( $v-t$ ) relation

let  $a = \frac{dv}{dt}$

$$\frac{dv}{dt} = -9.27 v^{\frac{1}{2}}$$

$$\int_{25}^v \frac{dv}{v^{\frac{1}{2}}} = -9.27 \int_0^t dt$$

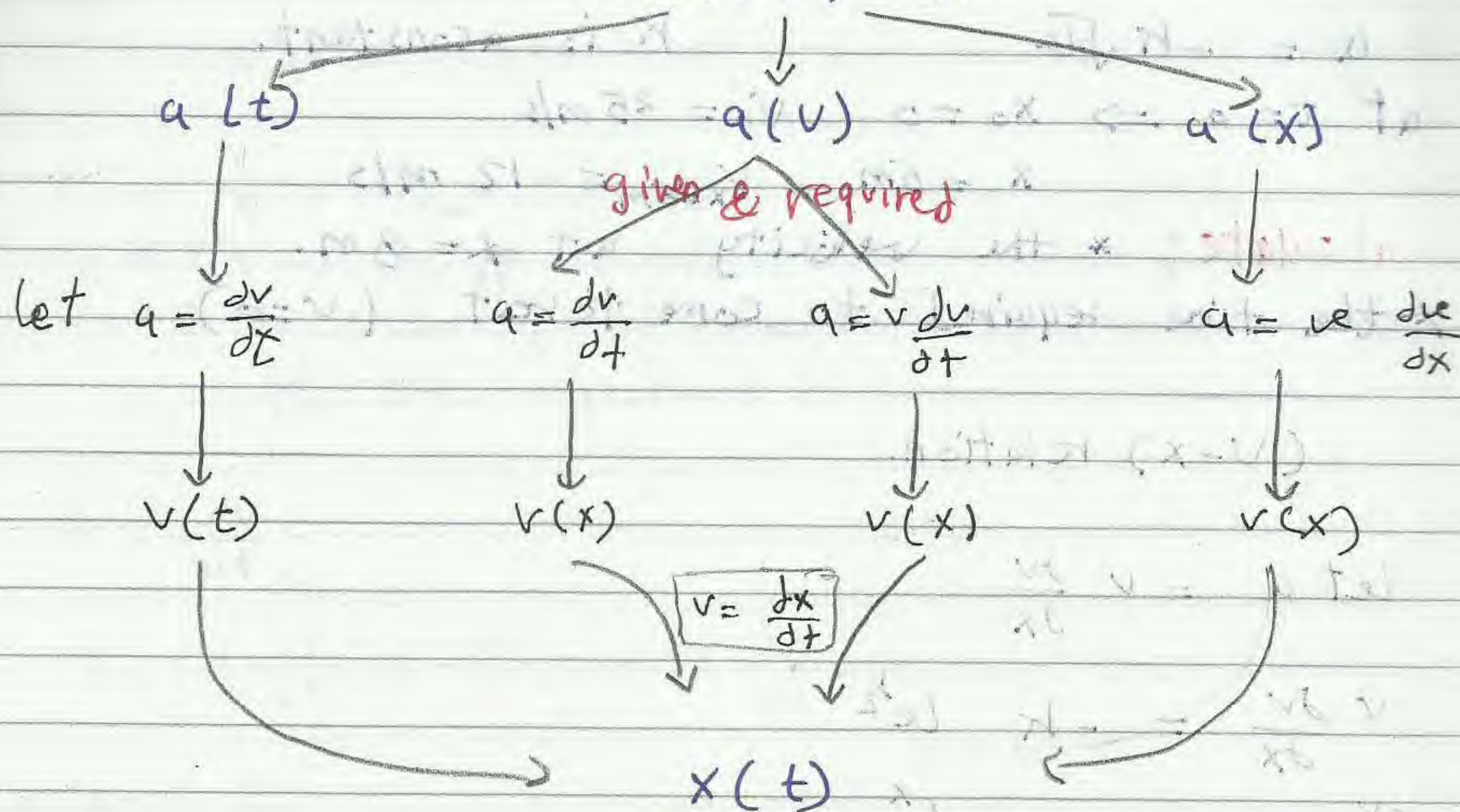
$$2|\sqrt{v}|_{25}^v = -9.27 t$$

$$2(\sqrt{v} - 5) = -9.27 t$$

at  $v=0 \Rightarrow t = \frac{10}{9.27} \text{ s}$



# Acceleration



For constant acceleration.

$$v = v_0 + at$$

$$d = v_0 t + \frac{1}{2} at^2$$

$$v^2 = v_0^2 + 2ad$$



# Curvilinear motion

Position:  $\vec{r}(t) = x(t)\hat{i} + y(t)\hat{j} + z(t)\hat{k}$

Velocity:  $\vec{v}(t) = \frac{d\vec{r}}{dt}$

$$\vec{v} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} + \frac{dz}{dt}\hat{k}$$

$$= \dot{x}\hat{i} + \dot{y}\hat{j} + \dot{z}\hat{k}$$

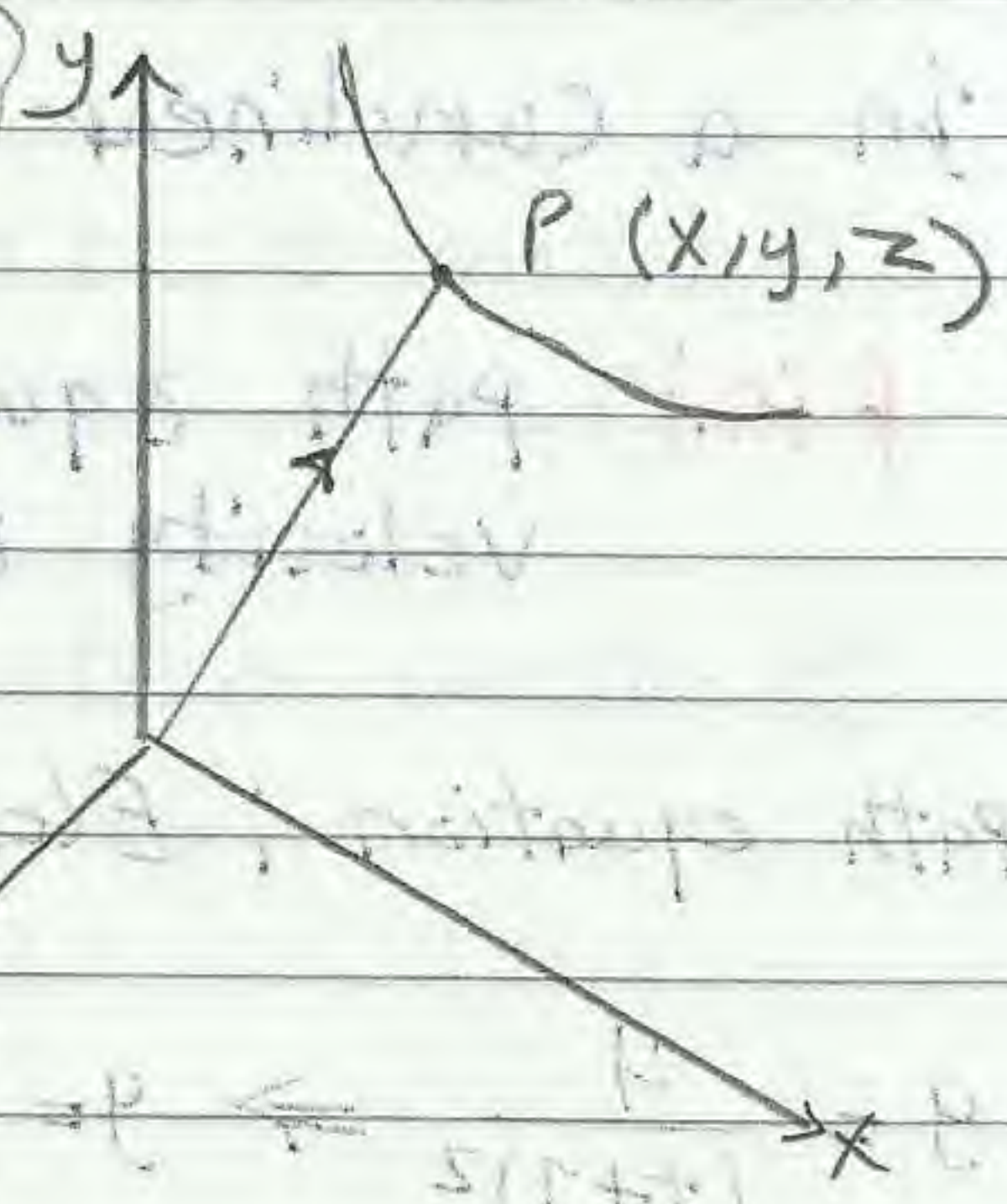
Speed = magnitude of velocity

$$= \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$$

Acceleration:  $\vec{a}(t) = \frac{d\vec{v}}{dt}$

$$\vec{a} = \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j} + \frac{d^2z}{dt^2}\hat{k} = \ddot{x}\hat{i} + \ddot{y}\hat{j} + \ddot{z}\hat{k}$$

magnitude of acceleration =  $\sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}$





In a curvilinear motion  $x = (t+1)^2$   $y = 4(t+1)^{-2}$

Find: path equation.

velocity & acceleration at  $t=0$

path equation: Eliminate  $t$  between  $x, y$

$$y = \frac{4}{(t+1)^2} \Rightarrow y = \frac{4}{x} \Rightarrow \boxed{xy = 4}$$

$$x = (t+1)^2$$

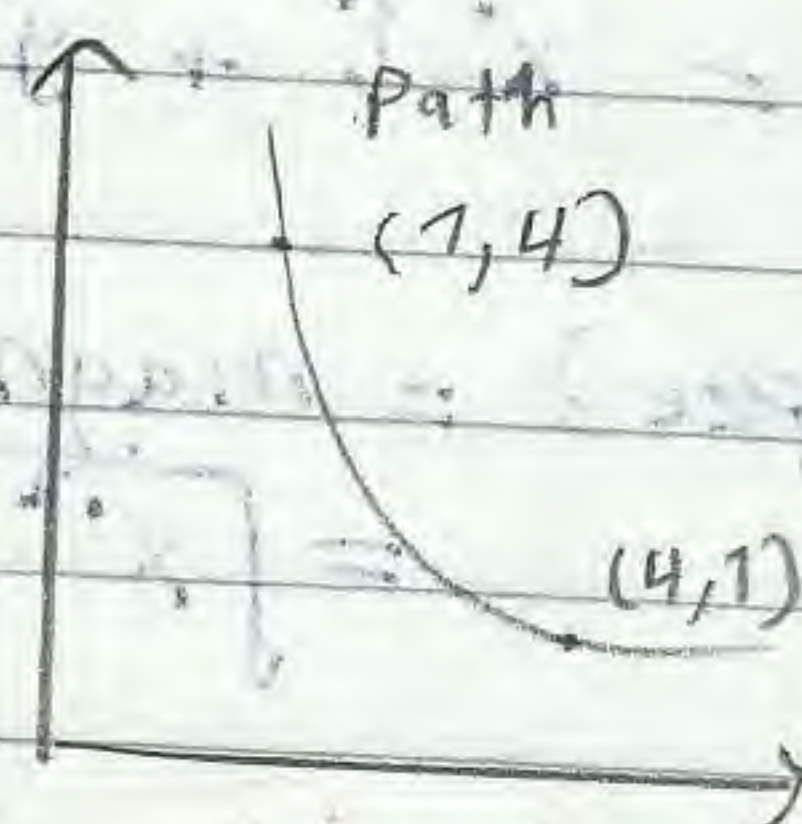
$$y = 4(t+1)^{-2}$$

$$\dot{x} = \frac{dx}{dt} = 2(t+1)$$

$$\dot{y} = \frac{dy}{dt} = -8(t+1)^{-3}$$

$$\ddot{x} = \frac{d^2x}{dt^2} = 2$$

$$\ddot{y} = \frac{d^2y}{dt^2} = 24(t+1)^{-4}$$



at  $t=0$

$$\dot{x} = 2 \text{ m/s}$$

$$\dot{y} = -8 \text{ m/s}^2$$

$$\ddot{x} = 2 \text{ m/s}^2$$

$$\ddot{y} = 24 \text{ m/s}^2$$

$$\vec{v}_0 = \dot{x}_0 \hat{i} + \dot{y}_0 \hat{j} = 2 \hat{i} - 8 \hat{j}$$

$$\vec{a}_0 = \ddot{x}_0 \hat{i} + \ddot{y}_0 \hat{j} = 2 \hat{i} + 24 \hat{j}$$

$$\tan \theta = \frac{y}{x}$$



Problem (11.95) Page 653

A particle in a curvilinear

$$\vec{r} = A(\cos t + t \sin t) \hat{i} + A(\sin t - t \cos t) \hat{j}$$

$$x = A(\cos t + t \sin t) \quad y = A(\sin t - t \cos t)$$

Calculate the time at which the position vector & acceleration vector are i) perpendicular.

ii) parallel.

$$\dot{x} = A(-\sin t + \sin t + t \cos t) \quad \dot{y} = A(\cos t - \cos t + t \sin t)$$

$$= A t \cos t \quad = A t \sin t$$

$$\ddot{x} = A(\cos t - t \sin t) \quad \ddot{y} = A(\sin t + \cos t)$$

case (1)  $\vec{r} \perp \vec{a} \Rightarrow \vec{r} \cdot \vec{a} = 0$

$$x \ddot{x} + y \ddot{y} = 0$$

$$A^2 (\cos t + t \sin t) (\cos t - t \sin t) + A^2 (\sin t - t \cos t) (\sin t + \cos t) = 0$$

$$A^2 [\cos^2 t - t^2 \sin^2 t + \sin^2 t - t^2 \cos^2 t] = 0$$

$$1 - t^2 = 0 \Rightarrow t = 1 \text{ or } t = -1$$

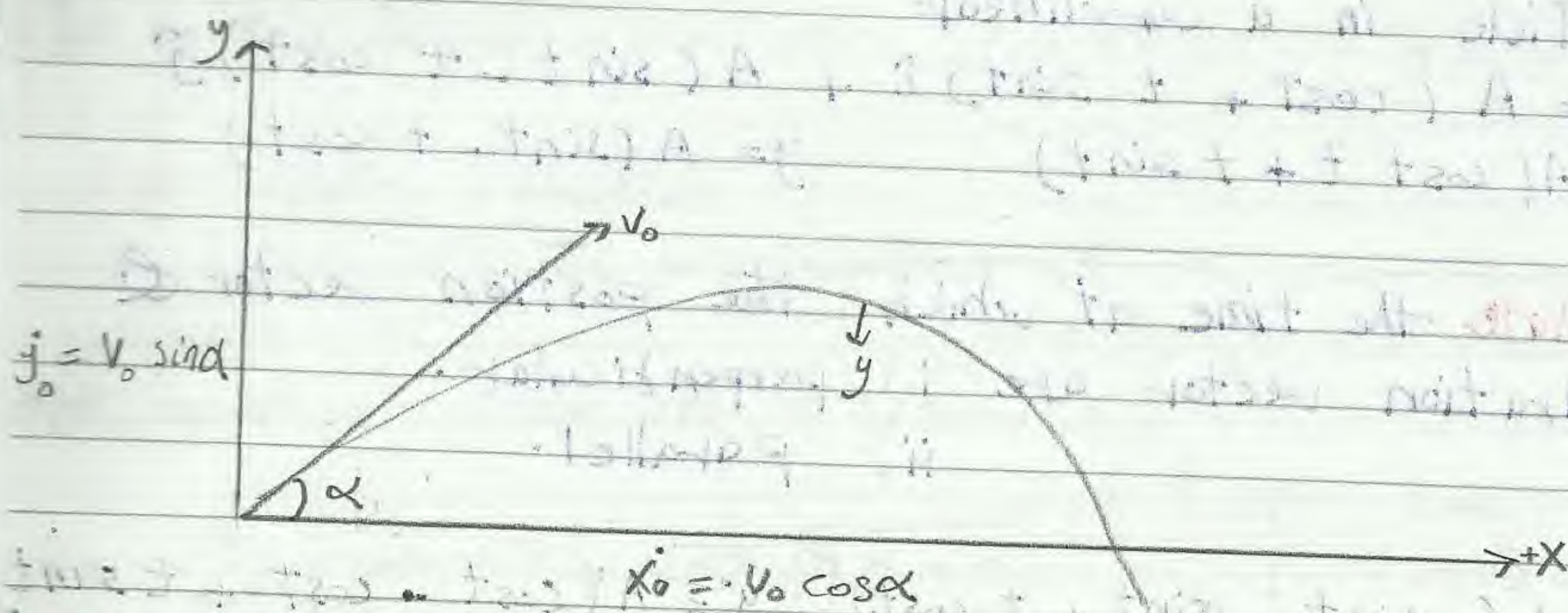
case (2)  $\vec{r} \parallel \vec{a} \Rightarrow \vec{r} \times \vec{a} = 0$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ x & y & 0 \\ \ddot{x} & \ddot{y} & 0 \end{vmatrix} = 0$$

$$x \ddot{y} - y \ddot{x} = 0 \Rightarrow t = 0$$



# Curvilinear motion (projectiles)



- \* 1 - Origin at the initial point.
- 2 - x-axis is the direction of projectile.
- 3 - y-axis is vertically upward.

Accelerations:

$$a_x = \ddot{x} = 0 \quad (1)$$

$$a_y = \ddot{y} = -g \quad (2)$$

$$v_x = \dot{x} = C_1$$

$$v_y = \dot{y} = -gt + C_2$$

at  $t=0$   $\dot{x}_0 = v_0 \cos \alpha$

$$\dot{y} = v_0 \sin \alpha$$

$$v_x = \dot{x} = v_0 \cos \alpha \quad (3)$$

$$v_y = \dot{y} = v_0 \sin \alpha - gt \quad (4)$$

$$x = (v_0 \cos \alpha)t + C_3$$

$$y = (v_0 \sin \alpha)t - \frac{1}{2}gt^2 + C_4$$

at  $t=0 \Rightarrow x_0 = 0, y_0 = 0$

$$x = (v_0 \cos \alpha)t \quad (5)$$

$$y = (v_0 \sin \alpha)t - \frac{1}{2}gt^2 \quad (6)$$

$$y^2 = (v_0 \sin \alpha)^2 - 2gy \quad (7)$$



path equation:

⑥ is image, ⑤ is T color

$$y = x \tan \alpha - \frac{g x^2}{2 v^2 \cos^2 \alpha}$$

⑧

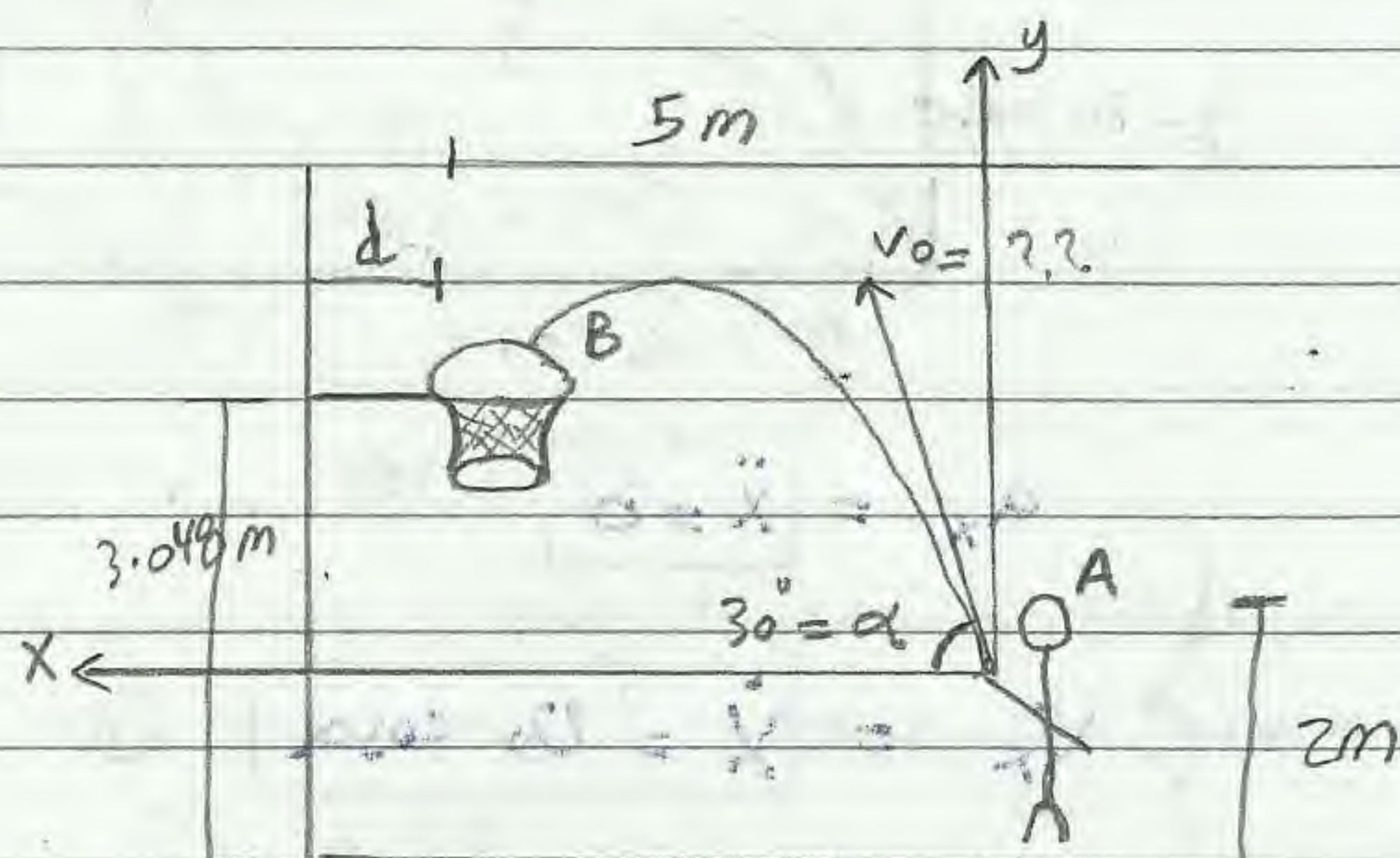
problem (11.110) page 656

calculate  $v_0$

case ①  $d = 228 \text{ mm}$

case ②  $d = 430 \text{ mm}$

calculate  $t_{A \rightarrow B}$



case ①  $d = 228 \text{ mm} = 0.228 \text{ m}$

at B  $x_B = 5 - d$   $y_B = 3.048 - 2$   $\alpha = 30^\circ$

$$y = x \tan \alpha - \frac{g x^2}{2 v^2 \cos^2 \alpha}$$

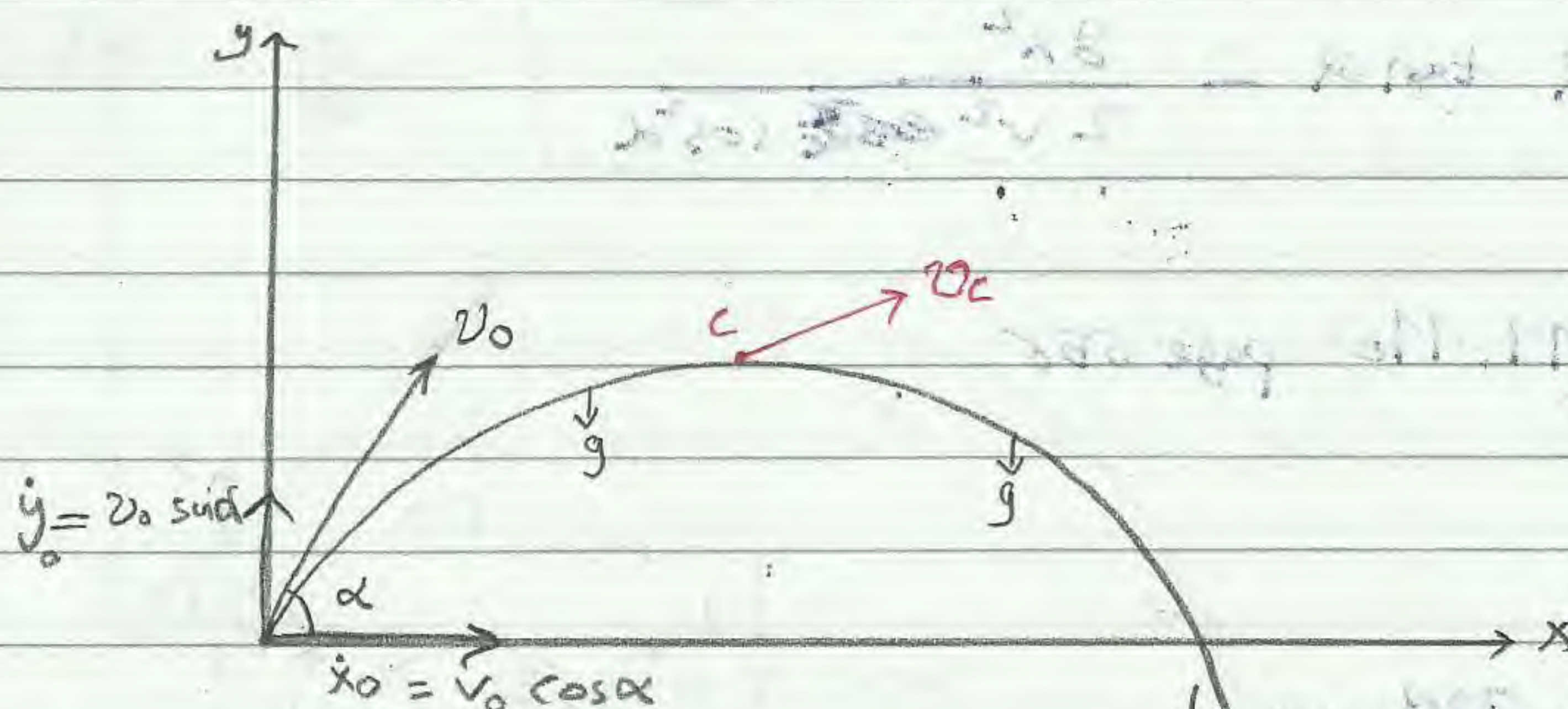
$$1.048 = 4.772 \tan 30^\circ - \frac{9.8 (4.772)^2}{2 v_0^2 \cos^2 30^\circ}$$

$$v_0 = v_0$$

$$t_{A-B} = \frac{x}{v_0 \cos \alpha} = \frac{4.772}{v_0 \cos 30^\circ}$$



## projectiles



$$a_x = \ddot{x} = 0 \quad (1)$$

$$a_y = \ddot{y} = -g \quad (2)$$

$$v_x = \dot{x} = v_0 \cos \alpha \quad (3)$$

$$v_y = \dot{y} = v_0 \sin \alpha - g t \quad (4)$$

$$x = (v_0 \cos \alpha) t \quad (5)$$

$$y = v_0 \sin \alpha t - \frac{1}{2} g t^2 \quad (6)$$

$$v^2 = v_0^2 + 2 a d \rightarrow$$

$$\dot{y}^2 = (v_0 \sin \alpha)^2 - 2 g y \quad (7)$$

Path equation

path equation

$$y = x \tan \alpha - \frac{g x^2}{2 v_0^2 \cos^2 \alpha} \quad (8)$$

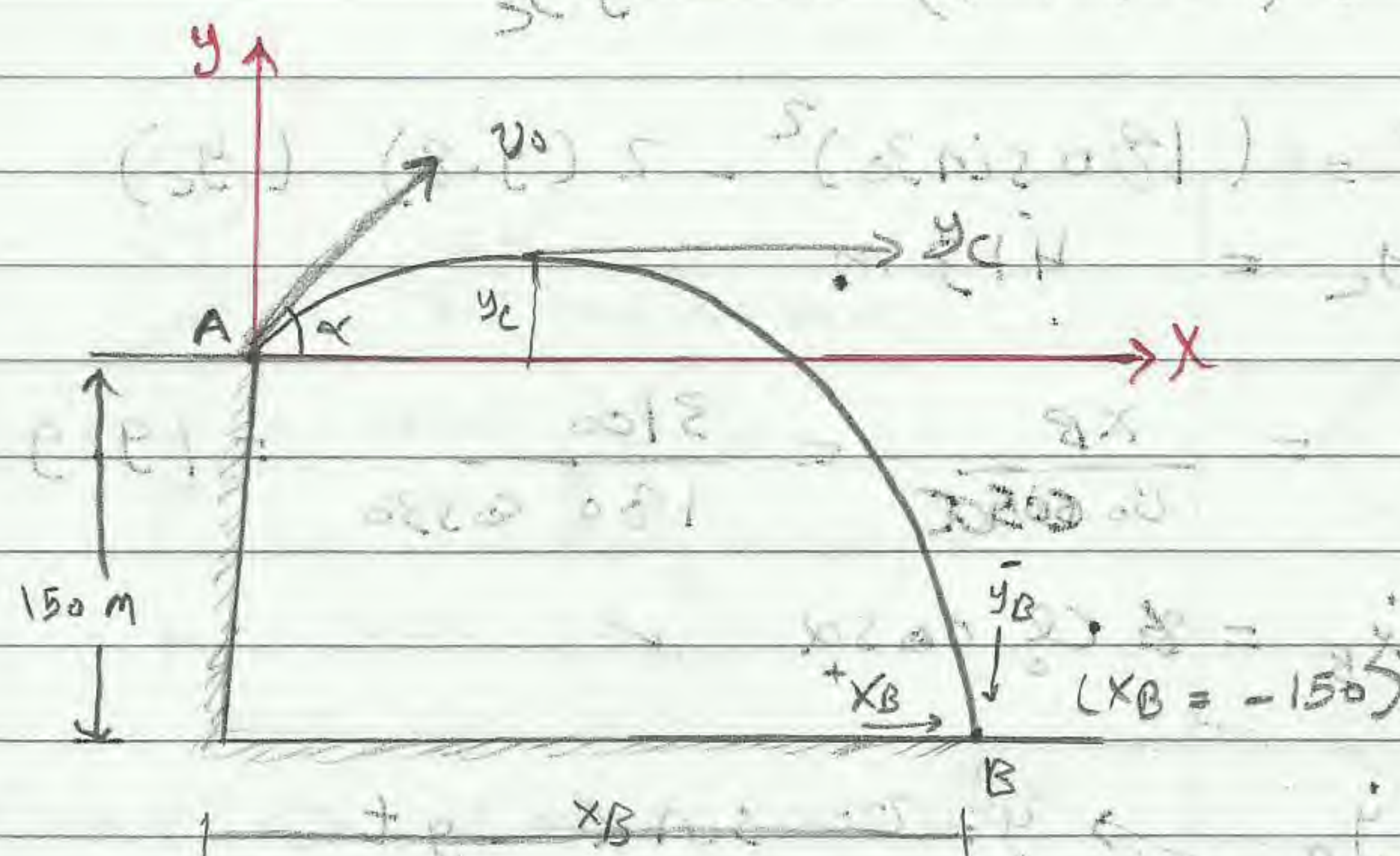
لو عاير او صيني T تشغل من 6 → 7

لو اراش 3 بيت اراج من 8  $[x, y, v_0, \alpha]$



\* Example (11.7) page 548

مسألة أمثلة 11.7



Given:  $v_0 = 180 \text{ m/s}$ ,  $\alpha = 30^\circ$

find i)  $x_B = ??$ , ii) max height above the ground.

ii)  $t_{A \rightarrow B} = ??$ ,  $\vec{v}_B = ??$

$x, y$ , origin  $\leftarrow$  projectile

$$y_B = -150 \text{ m}$$

$$y = x \tan \alpha - \frac{g x^2}{2 v_0^2 \cos^2 \alpha}$$

$$-150 = x_B \tan 30^\circ - \frac{9.8 x_B^2}{2 (180)^2 \cos^2 30^\circ}$$

$$\left[ \frac{9.8}{2 (180)^2 \cos^2 30^\circ} \right] x_B^2 - [\tan 30^\circ] x_B - 150 = 0$$

لو صارت  
ألف

$$x_B = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

$$x_B = 3100 \text{ m}$$

$$x_B = -ve \text{ } x$$

ii) At max height(c)  $\dot{y}_c = 0$

$$\begin{cases} v_0 \sin \alpha - g t_{A \rightarrow c} = 0 \Rightarrow t_{A \rightarrow c} = \frac{v_0 \sin \alpha}{g} = 9.18 \text{ s} \\ y_c = (v_0 \sin \alpha) t_{A \rightarrow c} - \frac{1}{2} (9.8) t_{A \rightarrow c}^2 \approx 493 \text{ m} \end{cases}$$

above ground add (150 m)



or

$$y_c^2 = (v_0 \sin \alpha)^2 - 2gy_c$$

$$0^2 = (180 \sin 30)^2 - 2(9.8)(y_c)$$

$$\Rightarrow y_c = 413 \text{ m}$$

$$\text{iii)} \quad t_{AB} = \frac{x_B}{v_0 \cos \alpha} = \frac{3100}{180 \cos 30} = 19.9 \text{ s}$$

$$\vec{v}_B \quad x_B = v_0 \cos \alpha \quad \checkmark$$

$$y_B \quad \begin{cases} y = v_0 \sin \alpha - gt \\ \text{or} = 180 \sin 30 - 9.8(19.9) \\ y^2 = (v_0 \sin \alpha)^2 - 2gy \end{cases}$$

گفتی باید صاف از این تار است

$$= (180 \sin 30)^2 - 2(9.8)(-150)$$

مثال ۱۱

س. ۱

س. ۲

$$0 = 0.21 = gX [\sin \alpha] - \frac{gX}{\cos \alpha} \left[ \frac{0.21}{\sin \alpha} - \cos \alpha \right]$$

$$0.21 = gX$$

$$X = \frac{0.21}{g} = \frac{0.21}{9.8} = 0.021 \text{ m}$$

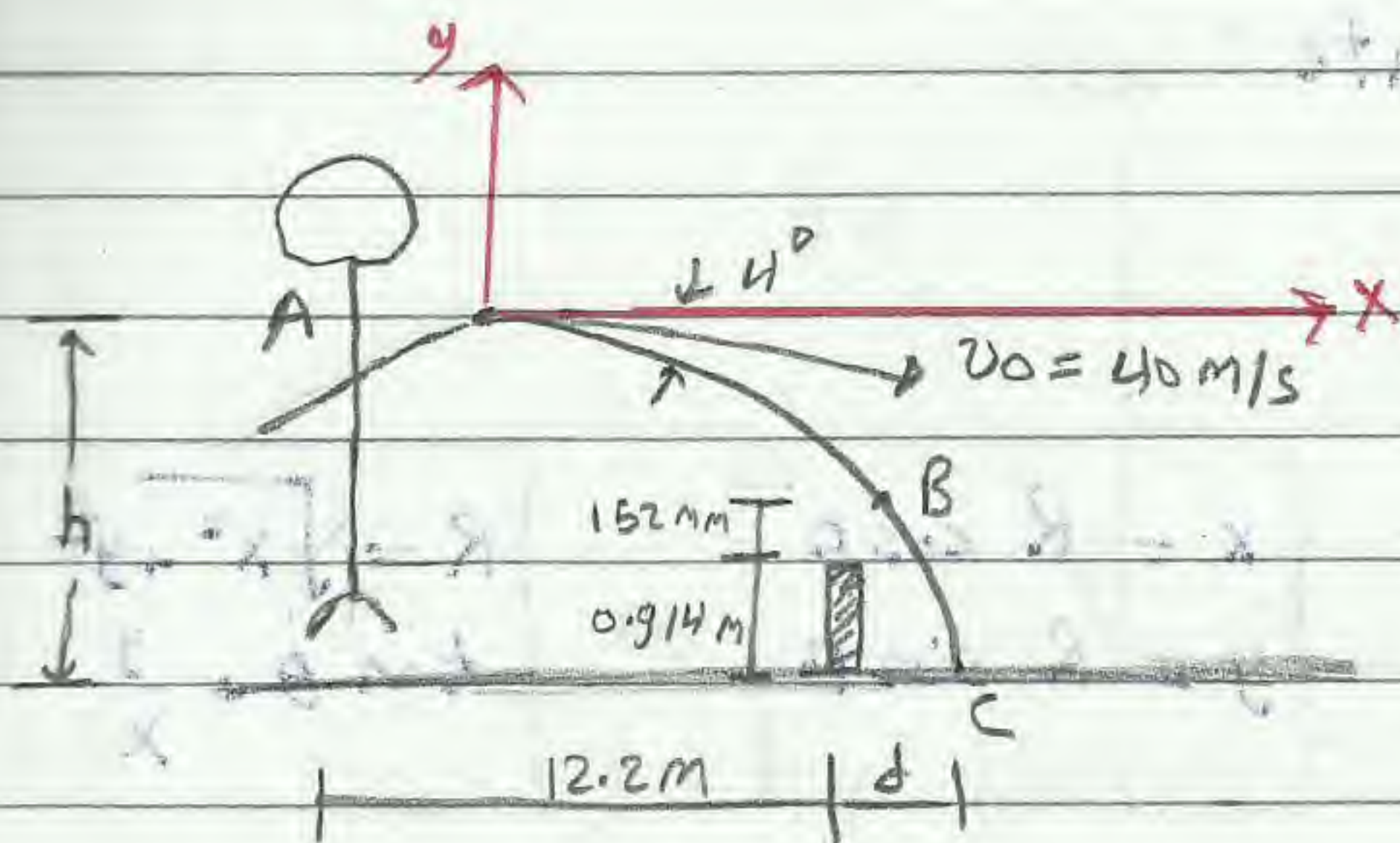
$$X = 0.021 \text{ m}$$

مثال ۱۲ (مثال ۱۱ را با ۱۲)

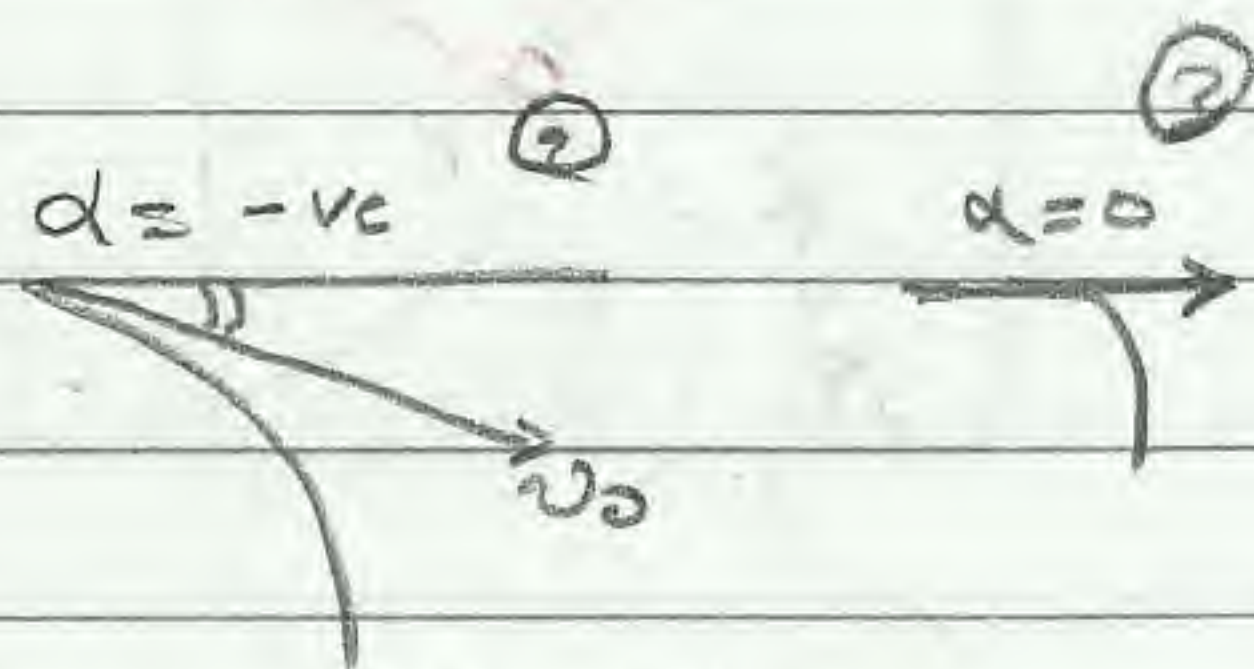
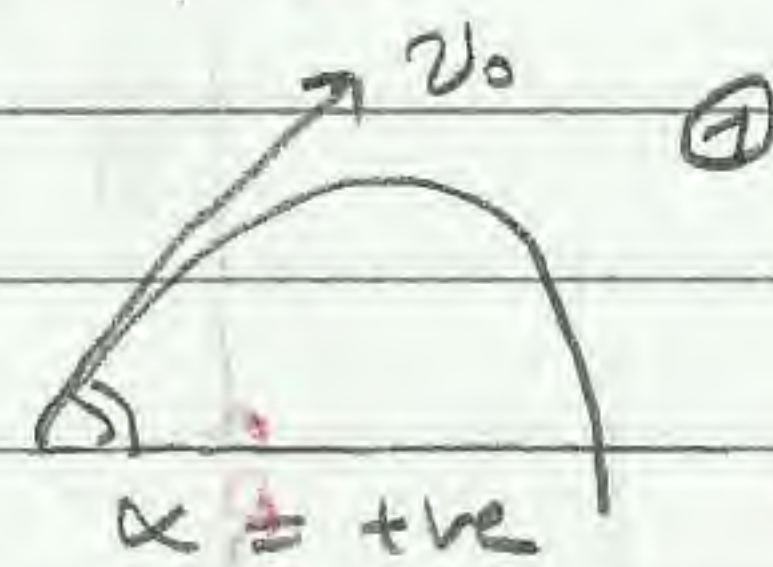
$$\begin{aligned} \text{مثال ۱۲} & \rightarrow \text{مثال ۱۱} \rightarrow \text{مثال ۱۲} \rightarrow \text{مثال ۱۳} \\ \text{مثال ۱۳} & \rightarrow \text{مثال ۱۴} \rightarrow \text{مثال ۱۵} \rightarrow \text{مثال ۱۶} \end{aligned}$$



# Problem (11.106) Page 698



\* Note



Given  $v_0 = 40 \text{ m/s}$   $\alpha = -4^\circ$

Find  $h = ??$  ,  $d = ??$

$$y = x \tan \alpha - \frac{g x^2}{2 v_0^2 \cos^2 \alpha}$$

at B

$$y_B = 12.2 \tan(-4^\circ) - \frac{9.8 (12.2)^2}{2 (40)^2 \cos^2(-4^\circ)}$$

$$y_B = -1.3 \text{ m}$$

$$h = 0.914 + 0.152 + |y_B| = 2.4 \text{ m}$$

at C  $y_c = -2.4 = x_c \tan(-4^\circ) - \frac{9.8 x_c^2}{2 (40)^2 \cos^2(-4^\circ)}$

$$d = x_c - 12.2 = \sqrt{\dots}$$

Assignment: Find  $v_B$  ,  $v_C$

$$\vec{v}(\vec{r} + \vec{r}_s) + \vec{r}(\vec{v} - \vec{v}_s) = \vec{0}$$

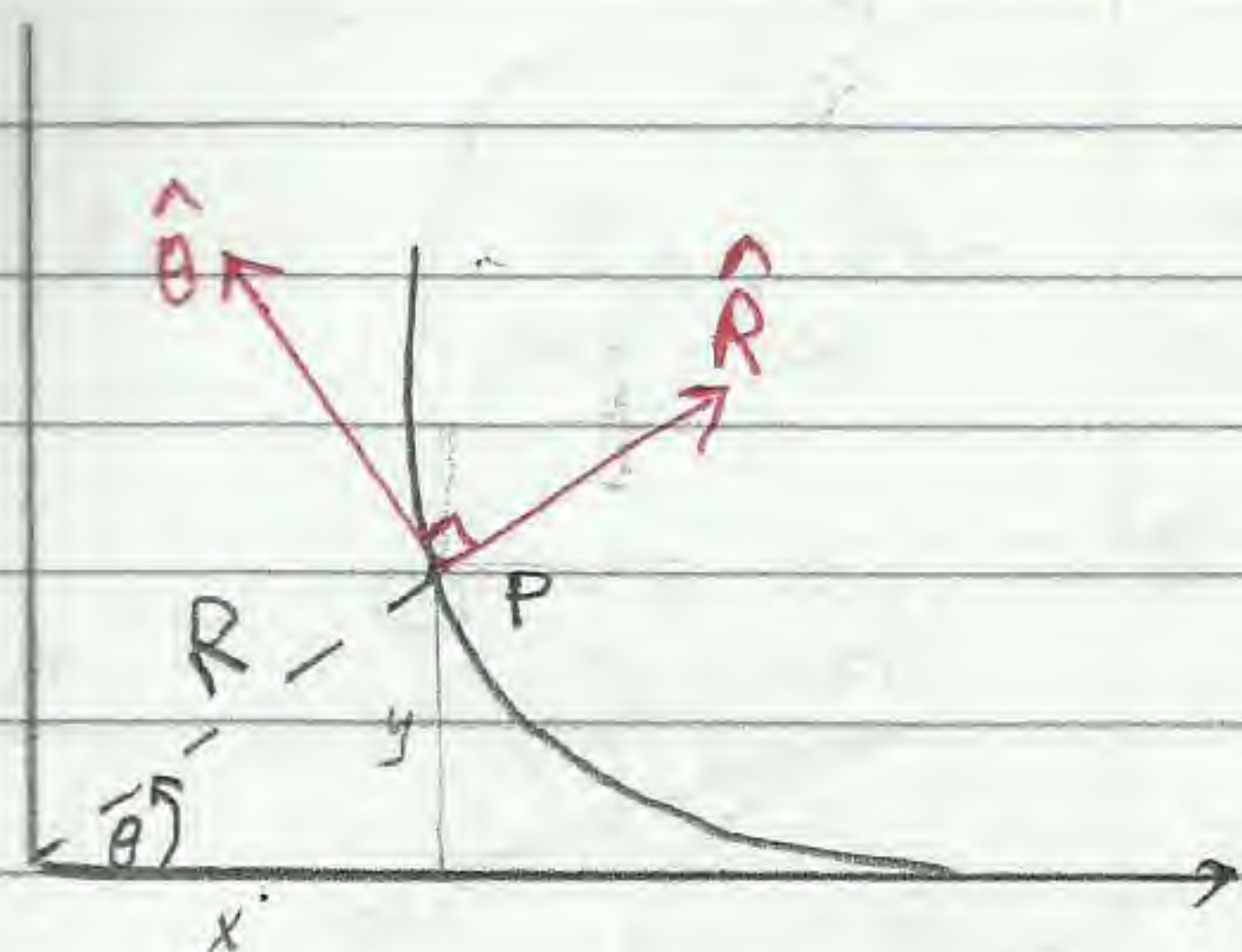
$$\vec{v} = \frac{\vec{r}}{r} \frac{dr}{dt} + \vec{r} \frac{d\theta}{dt}$$



# lect. 3

polar & cylindrical coordinate.

planer polar coordinate:



$$\begin{aligned} x &= R \cos \theta \\ y &= R \sin \theta \end{aligned}$$

$\Leftrightarrow$

$$\begin{aligned} R &= \sqrt{x^2 + y^2} \\ \tan \theta &= \frac{y}{x} \end{aligned}$$

Coordinates

Radial distance =  $R$

Transverse angle =  $\theta$

Unit vectors

Radial unit vector =  $\hat{R}$

Transverse unit vector =  $\hat{\theta}$

position:  $\vec{r} = R \hat{R}$

velocity:  $\vec{v} = \frac{d\vec{r}}{dt} = \dot{R} \hat{R} + R \dot{\theta} \hat{\theta}$

$$\vec{v} = \dot{R} \hat{R} + (R \dot{\theta}) \hat{\theta}$$

$\dot{R}$  = Radial velocity

$R \dot{\theta}$  = transverse velocity

$$\text{Speed} = \sqrt{V_R^2 + V_\theta^2}$$

Acceleration:  $\vec{a} = \frac{d\vec{v}}{dt} = \ddot{R} \hat{R} + \dot{R} \dot{\theta} \hat{R} + (\dot{R} \dot{\theta}) \hat{\theta} + (R \ddot{\theta}) \hat{\theta} + (R \dot{\theta}) \dot{\hat{\theta}}$

$$\vec{a} = (\ddot{R} - R \dot{\theta}^2) \hat{R} + (2 \dot{R} \dot{\theta} + R \ddot{\theta}) \hat{\theta}$$

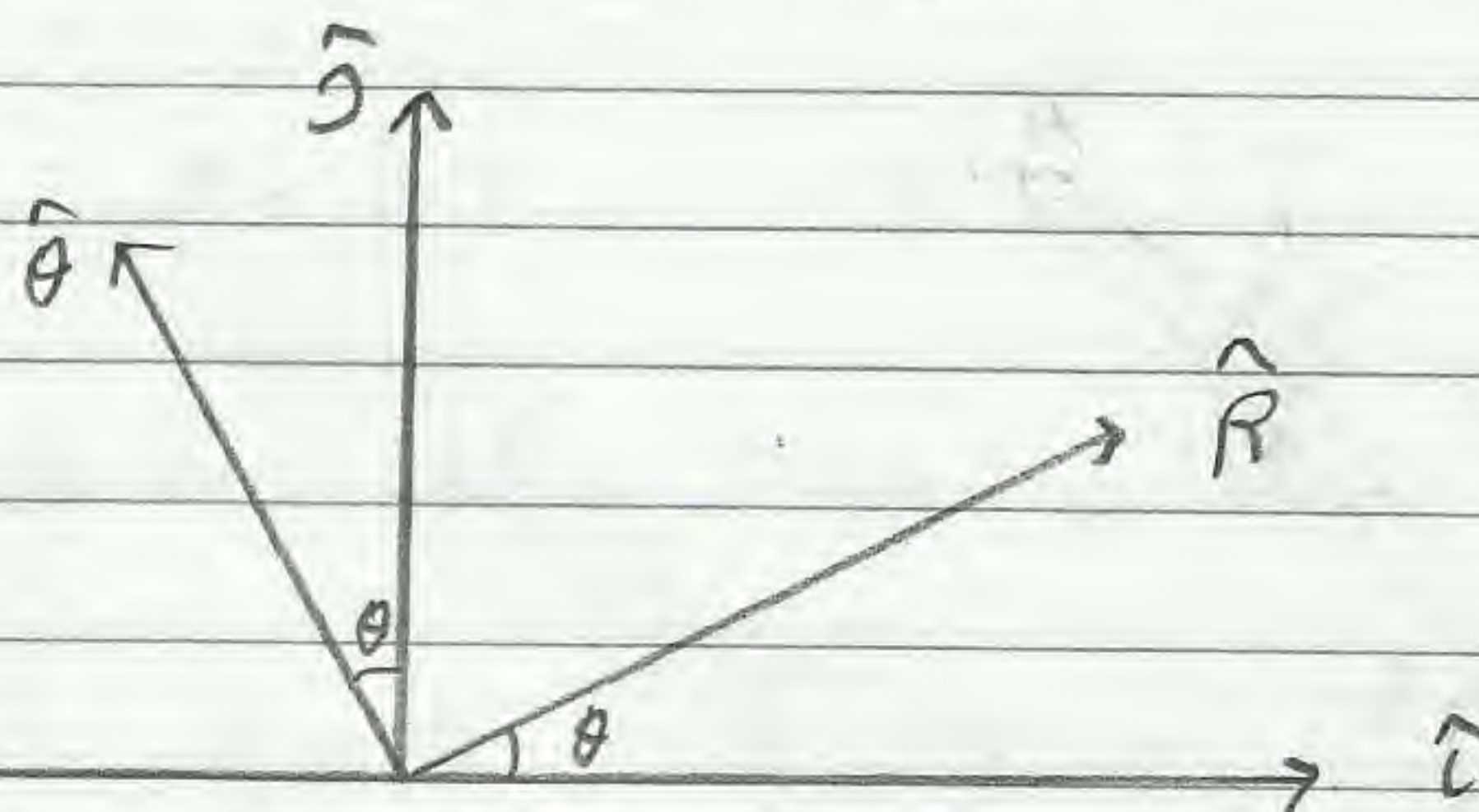
$a_R$  = Radial acceleration

$a_\theta$  = transverse acceleration

$$a = \sqrt{a_R^2 + a_\theta^2}$$



Time derivative of unit vector:  $\hat{r}$  and  $\hat{\theta}$



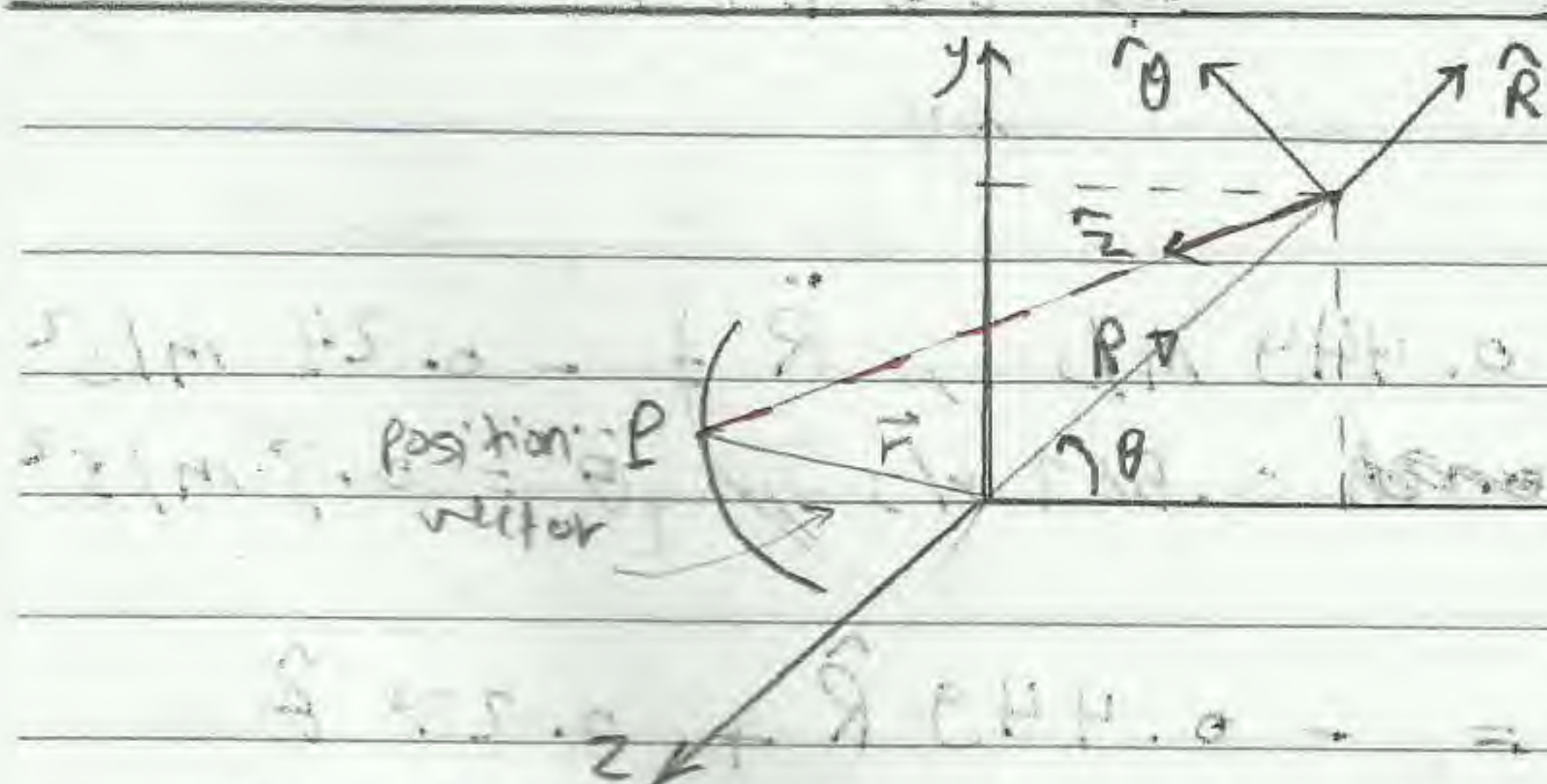
$$\hat{r} = (\cos \theta) \hat{i} + (\sin \theta) \hat{j}$$

$$\hat{\theta} = -(\sin \theta) \hat{i} + (\cos \theta) \hat{j}$$

$$\dot{\hat{r}} = \frac{d\hat{r}}{dt} = (-\sin \theta) \dot{\theta} \hat{i} + (\cos \theta) \dot{\theta} \hat{j} = (\dot{\theta}) \hat{\theta}$$

$$\dot{\hat{\theta}} = \frac{d\hat{\theta}}{dt} = (-\cos \theta) \dot{\theta} \hat{i} - (\sin \theta) \dot{\theta} \hat{j} = -(\dot{\theta}) \hat{r}$$

space cylindrical coordinate:



coordinates

Radial distance =  $R$

Transverse angle =  $\theta$

Axial distance =  $z$

unit vectors

Radial unit vector =  $\hat{r}$

Transverse unit vector =  $\hat{\theta}$

Axial unit vector =  $\hat{k}$

Position:  $\vec{r} = R \hat{r} + z \hat{k}$

velocity:  $\vec{v} = (\dot{R}) \hat{r} + (R \dot{\theta}) \hat{\theta} + \dot{z} \hat{k}$

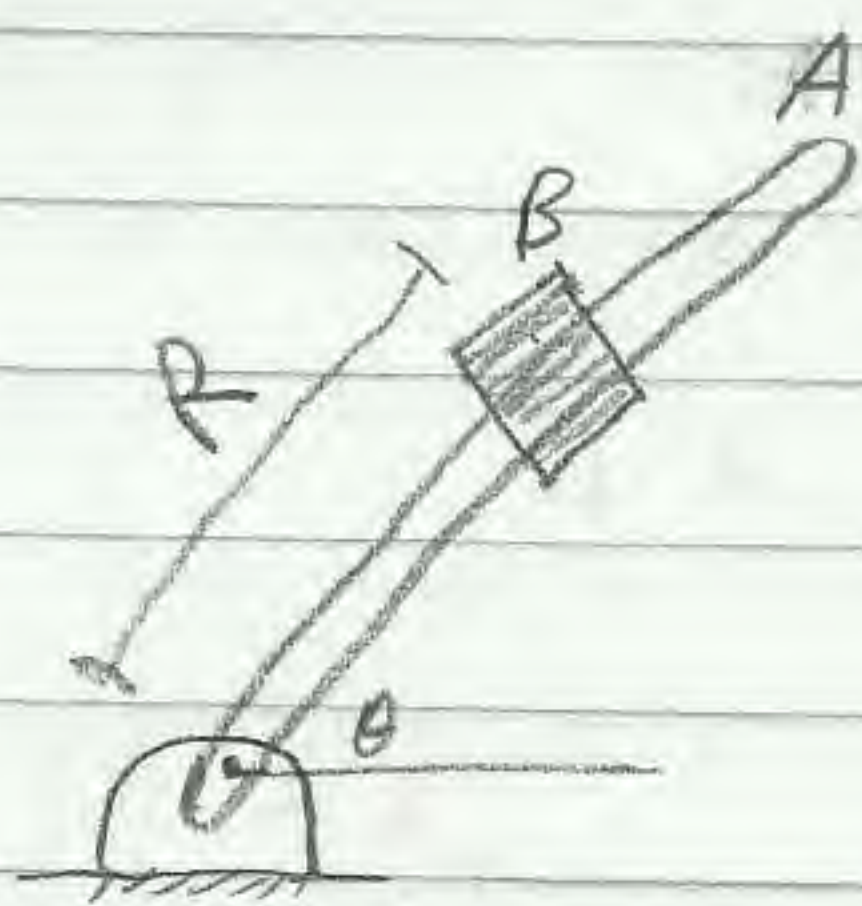
speed =  $\sqrt{v_R^2 + v_\theta^2 + v_z^2}$

Acceleration:  $\vec{a} = (\ddot{R} - R \dot{\theta}^2) \hat{r} + (2 \dot{R} \dot{\theta} + R \ddot{\theta}) \hat{\theta} + \ddot{z} \hat{k}$

$a = \sqrt{a_R^2 + a_\theta^2 + a_z^2}$



# Example (11-12) Page 669



$$R = 0.9 - 0.12 t^2$$

$$\theta = 0.15 t^2$$

calculate:  $\vec{v}$ ,  $\vec{a}$  at  $\theta = 30^\circ$

$$\dot{R} = -0.24 t$$

$$\dot{\theta} = 0.3 t$$

$$\ddot{R} = -0.24$$

$$\ddot{\theta} = 0.3$$

let  $\theta = 30^\circ$

$$(30^\circ) \left( \frac{\pi}{180} \right)^{3.14} = 0.15 t^2$$

$$\Rightarrow t = 1.87$$

at  $t = 1.87$

$$R = 0.48 \text{ m}, \quad \dot{R} = -0.449 \text{ m/s}, \quad \ddot{R} = -0.24 \text{ m/s}^2$$

$$\theta = 0.524 \text{ rad}, \quad \dot{\theta} = 0.561 \text{ r/s}, \quad \ddot{\theta} = 0.3 \text{ m/s}^2$$

$$\vec{v} = (\dot{R}) \hat{R} + (R \dot{\theta}) \hat{\theta} = -0.449 \hat{R} + 0.27 \hat{\theta}$$

$$\vec{a} = (\ddot{R} - R \dot{\theta}^2) \hat{R} + (2 \dot{R} \dot{\theta} + R \ddot{\theta}) \hat{\theta}$$

$$= -0.391 \hat{R} - 0.359 \hat{\theta}$$

$$\hat{R} = \cos \theta \hat{i} + \sin \theta \hat{j}$$

$$\hat{\theta} = -\sin \theta \hat{i} + \cos \theta \hat{j}$$

$$\vec{v} = \dot{R} \cos \theta \hat{i} + \dot{R} \sin \theta \hat{j} - R \dot{\theta} \sin \theta \hat{i} + R \dot{\theta} \cos \theta \hat{j}$$

$$\vec{a} = \ddot{R} \cos \theta \hat{i} + \ddot{R} \sin \theta \hat{j} - R \ddot{\theta} \sin \theta \hat{i} - R \ddot{\theta} \cos \theta \hat{j} - 2 \dot{R} \dot{\theta} \sin \theta \hat{i} + 2 \dot{R} \dot{\theta} \cos \theta \hat{j} - R \dot{\theta}^2 \cos \theta \hat{i} - R \dot{\theta}^2 \sin \theta \hat{j}$$



Problem (11.180) page 679

$$R = \frac{A}{t+1}$$

$$\theta = Bt$$

$$z = \frac{Ct}{t+1}$$

calculate  $\vec{v}$  &  $\vec{a}$  at  $t=0$ ,  $t \rightarrow \infty$

$$\dot{R} = \frac{-A}{(t+1)^2}$$

$$\dot{\theta} = B$$

$$\dot{z} = \frac{(t+1-t)}{(t+1)^2}$$

$$\ddot{R} = \frac{2A}{(t+1)^3}$$

$$\ddot{\theta} = 0$$

$$\ddot{z} = \frac{-2C}{(t+1)^3}$$

at  $t \rightarrow \infty$

$$R = 0, \dot{R} = 0, \ddot{R} = 0$$

$$\dot{\theta} = B, \ddot{\theta} = 0$$

$$\dot{z} = 0, \ddot{z} = 0$$

$$\vec{v} = 0, \vec{a} = 0$$

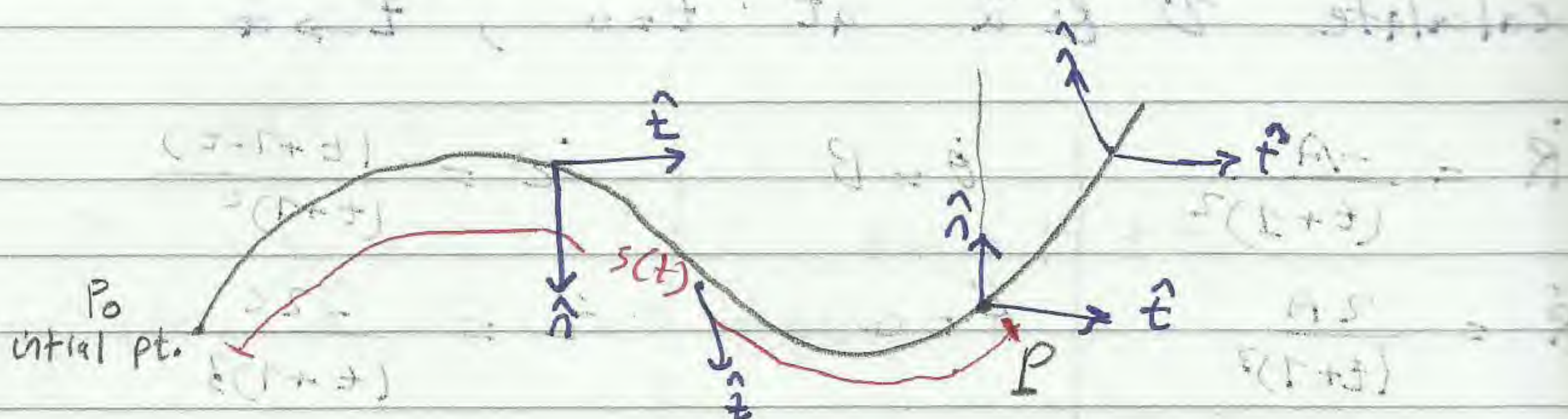


11/03/2013

## Curvilinear motion

### Intrinsic coordinates (Tangential & Normal Coordinates)

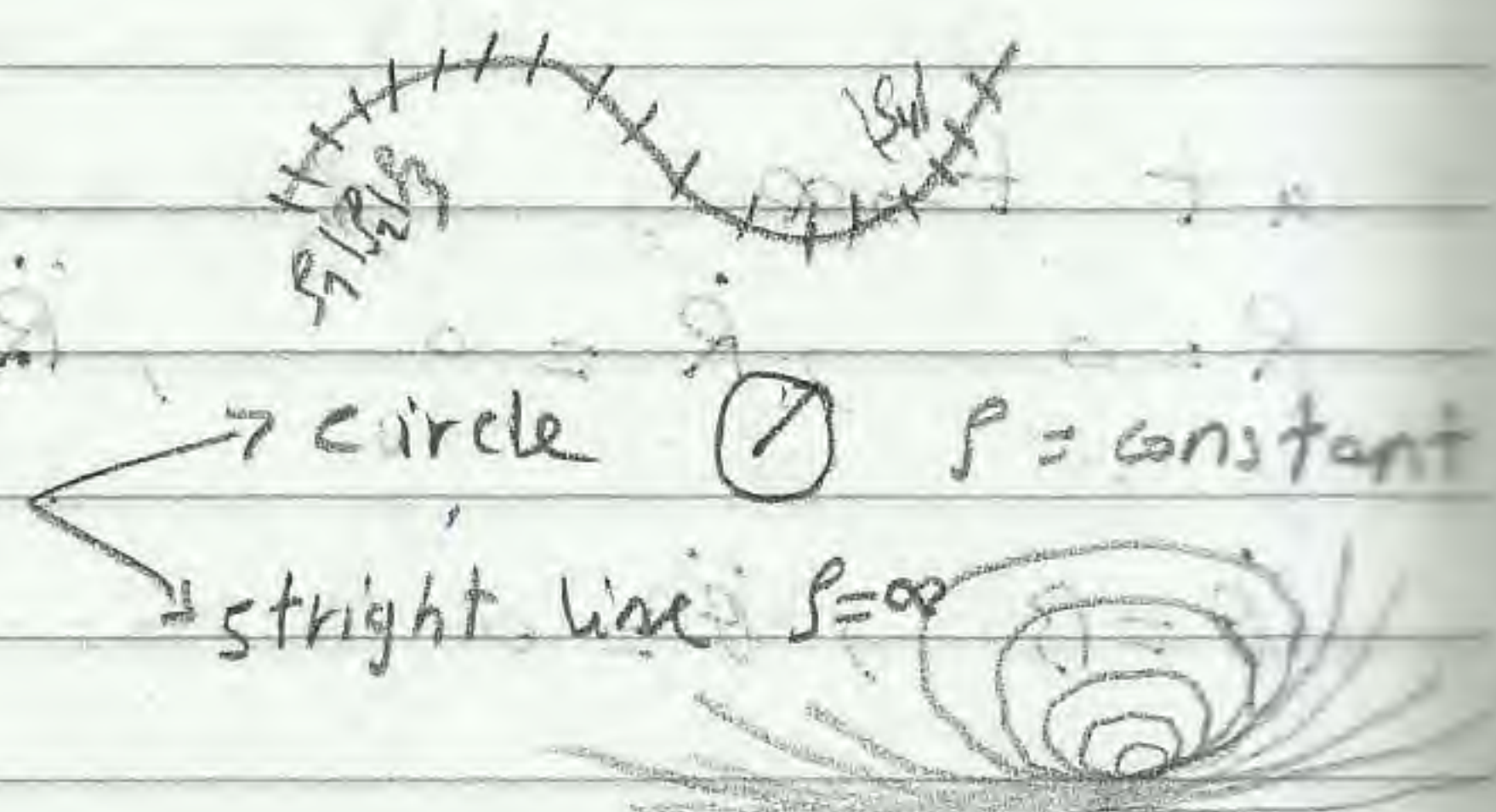
path curve is given



### Coordinates:

$s$  : Curved travel distance

$\rho$  : Radius of curvature



### Unit vectors:

$\hat{e}$  : tangential unit vector

$\hat{n}$  : normal unit vector  $\perp \hat{e}$  towards the centre

Position:  $\vec{r}$  not defined

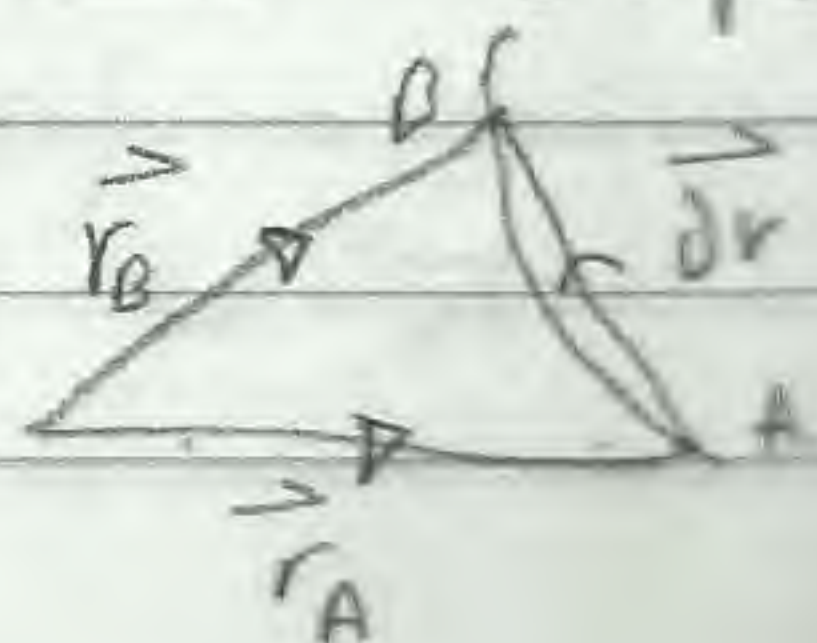
velocity:  $\vec{v} = \frac{d\vec{r}}{dt}$

$$\Delta s \rightarrow 0 \Rightarrow d\vec{r} = (ds) \hat{e}$$

$$\vec{v} = \frac{ds}{dt} \hat{e} \Rightarrow \boxed{v = (\dot{s}) \hat{e}}$$

velocity is tangent to path

magnitud = speed =  $v = \dot{s}$





Acceleration:  $\vec{a} = \frac{d\vec{v}}{dt} = (\dot{v})\hat{t} + v(\dot{\hat{t}})$

$$\vec{a} = \dot{v}\hat{t} + \dot{s}\left(\frac{v}{\rho}\right)\hat{n}$$

$$\vec{v} = v\hat{t} = (\dot{s})\hat{t}$$

$$\vec{a} = (\dot{v})\hat{t} + \dot{s}\left(\frac{v^2}{\rho}\right)\hat{n}$$

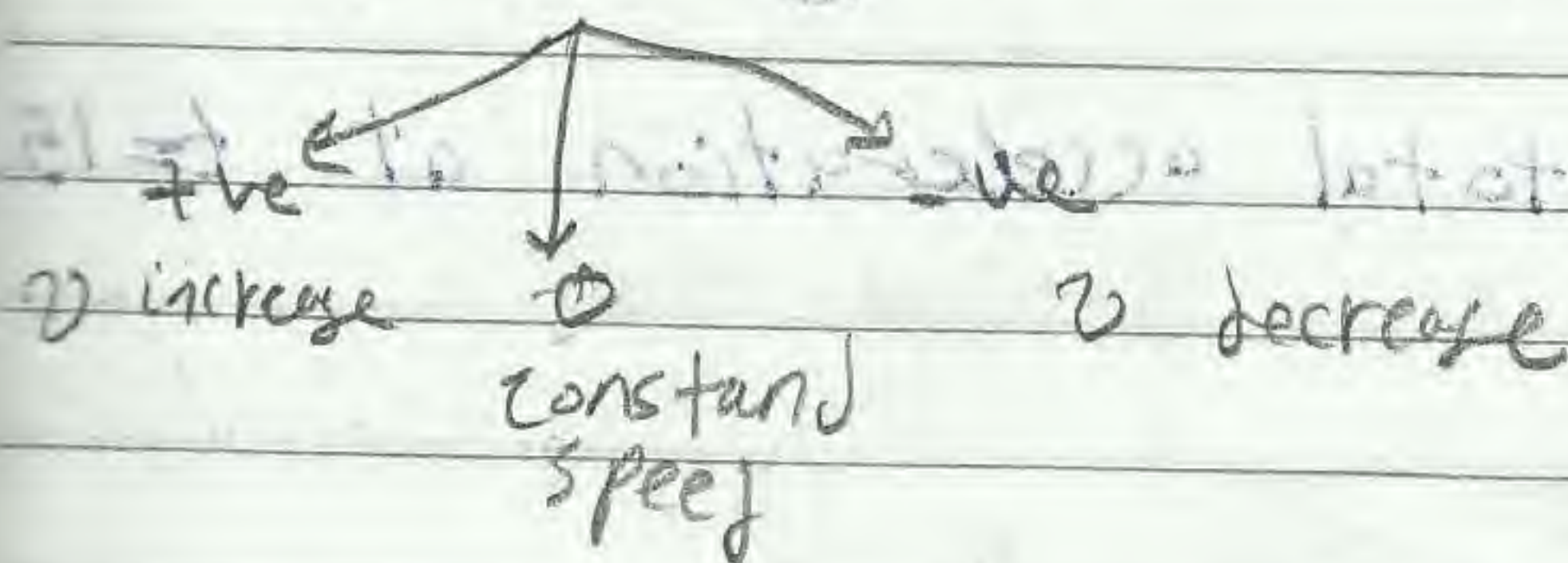
$$a_t = \dot{v} = \dot{s}$$

tangential acc.

$$a_n = \frac{v^2}{\rho}$$

Normal acc.

Time to travel distance  $s$



acceleration  $a_n = 0$  if  $\rho = \infty$

$$a_t = \frac{dv}{dt}$$

if  $a_t = \text{constant}$

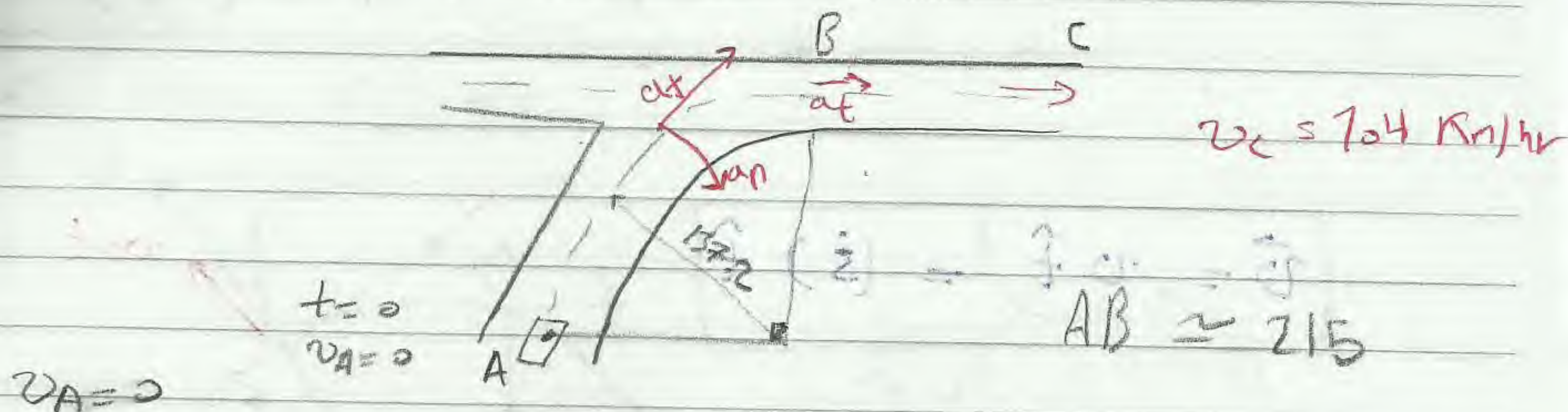
$$v = v_0 + a_t t$$

$$s = v_0 t + \frac{1}{2} a_t t^2$$

$$v^2 = v_0^2 + 2 a_t s$$



Problem (11.142) Page 673



$$a_t = \dot{v} = \text{constant} \quad \text{from } A \rightarrow C$$

$$v_C = 104 \text{ km/h} = 104 \times \frac{1000}{3600}$$

$$= 104 \times \frac{5}{18} = 28.8 \text{ m/s}$$

Calculate  $v_B = ??$  total acceleration at  $t = 15 \text{ s}$ ?

A  $\rightarrow$  B

$$v_B^2 = v_A^2 + 2 a_t s_{A \rightarrow B}$$

$$v_B^2 = 0^2 + 2 a_t \left( \frac{\pi (136.2)}{2} \right) \quad \text{--- (1)}$$

B  $\rightarrow$  C

$$v_C^2 = v_B^2 + 2 a_t s_{B \rightarrow C}$$

$$(28.8)^2 = v_B^2 + 2 a_t (97.4) \quad \text{--- (2)}$$

A  $\rightarrow$  C

$$(28.8)^2 = 0^2 + 2 a_t \left( \frac{\pi (136.2)}{2} + 97.4 \right)$$

$$a_t = 1.36 \text{ m/s}^2$$

A  $\rightarrow$  C (no  $a_r$ )  $\therefore$  const.  $a_t = 1.36$   
 sub in (1) or (2)

$$v_B = 24.2 \text{ m/s}$$



at

$$t = 15 \text{ s}$$

$$s(t) = s(0) + v(0)t + \frac{1}{2}at^2$$

$$s = v_0 t + \frac{1}{2} a t^2$$

$$= (0)(15) + \frac{1}{2} (7.36) (15)^2$$

$$s_{A \rightarrow B} = 810 \text{ m}$$

on curve

$$a_t = 7.36 \text{ m/s}^2$$

$$a_n = \frac{v^2}{\rho} = \frac{(0 + 7.36(15))^2}{137.2}$$

$$v = v_0 + at$$

$$\vec{a} = 7.36 \hat{t} + \dots \hat{n}$$

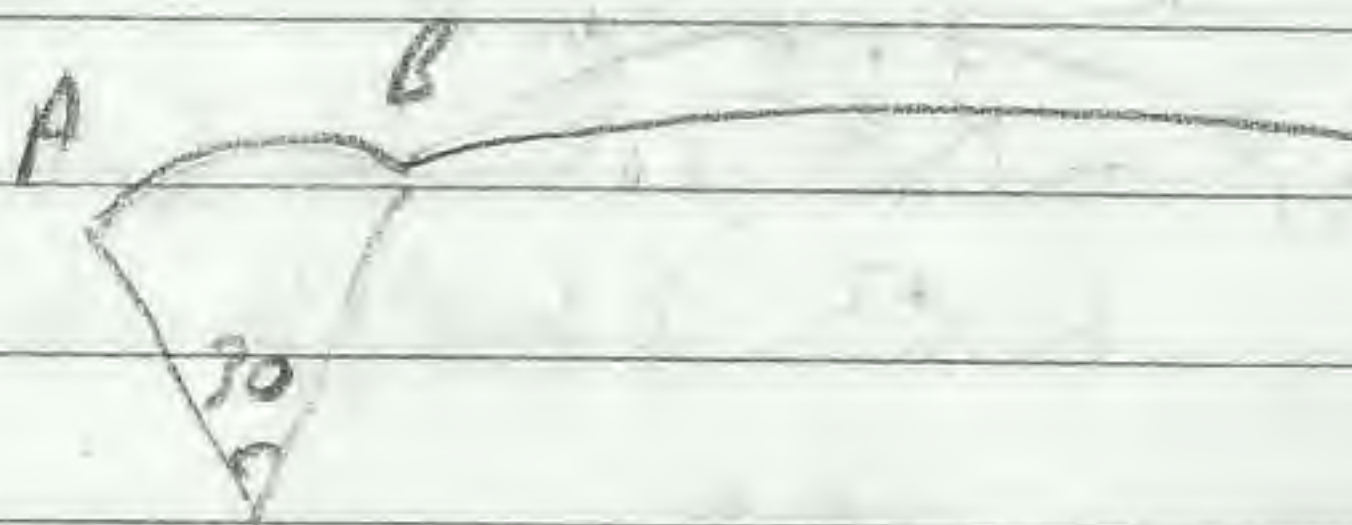
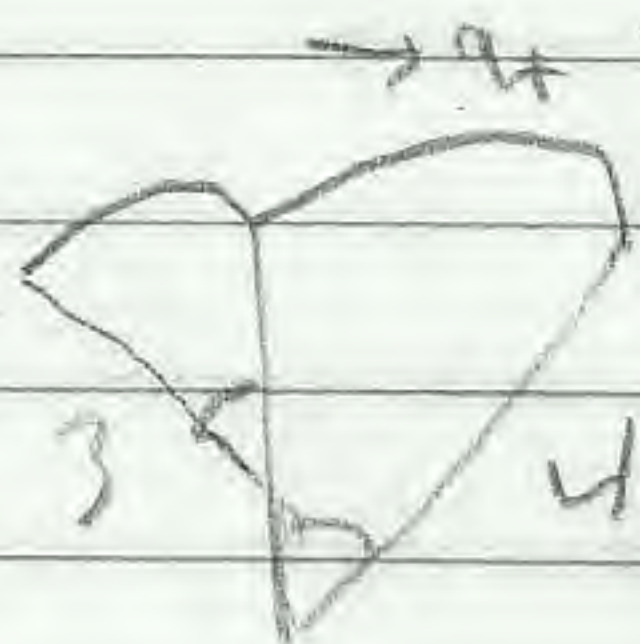
Just before B

Just after B

$$a_n = \frac{v_B^2}{137.2}$$

radius x centre angle

$$s_{A \rightarrow B} = r \left( \frac{\pi}{6} \right)$$





18/03/2013

polar

$$\vec{r} = (R) \hat{R} + (z) \hat{K}$$

$$\vec{v} = (\dot{R}) \hat{R} + (R\dot{\theta}) \hat{\theta} + (\dot{z}) \hat{K}$$

$$\vec{a} = (\ddot{R} - R\dot{\theta}^2) \hat{R} + (2\dot{R}\dot{\theta} + R\ddot{\theta}) \hat{\theta}$$

Intrinsic

$$\vec{v} = (v) \hat{t} = (\dot{s}) \hat{t}$$

$$\vec{a} = (\dot{v}) \hat{t} + \left(\frac{v^2}{\rho}\right) \hat{n}$$

problem (11.166) page 677

$$R = b(2 + \cos \pi t)$$

$$\theta = \pi t$$

calculate: ①  $\vec{v}_{2s}$ ,  $\vec{a}_{2s}$  at  $t=2s$ .  
② value of  $\theta$  for which  $v$  is max.

$$\dot{R} = -b\pi \sin(\pi t)$$

$$\ddot{R} = -b\pi^2 \cos(\pi t)$$

$$\dot{\theta} = \pi$$

$$\ddot{\theta} = 0$$

$$\text{at } t=2s \Rightarrow R=3b, \quad \dot{R}=0, \quad \ddot{R}=-b\pi^2$$
$$\dot{\theta}=\pi, \quad \ddot{\theta}=0$$

$$\vec{v}_{2s} = (\dot{R}) \hat{R} + (R\dot{\theta}) \hat{\theta} = (3b\pi) \hat{\theta}$$

$$\vec{a}_{2s} = (\ddot{R} - R\dot{\theta}^2) \hat{R} + (2\dot{R}\dot{\theta} + R\ddot{\theta}) \hat{\theta}$$
$$= (-b\pi^2 - 3b\pi^2) \hat{R} = -(4b\pi^2) \hat{R}$$



For max. & min speed.

$$v = \sqrt{\dot{R}^2 + (R\dot{\theta})^2} = \sqrt{b^2 \pi^2 \sin^2(\pi t) + b^2 \pi^2 (2 + \cos \pi t)^2}$$
$$= b \pi \sqrt{\sin^2 \pi t + \cos^2 \pi t + 4 + 4 \cos(\pi t)}$$

$$v = b \pi \sqrt{5 + 4 \cos \pi t}$$

method ①

$$\frac{dv}{dt} = 0 = \frac{b \pi (-4 \sin \pi t)}{2 \sqrt{5 + 4 \cos \pi t}}$$

method ②

$$v_{\min} \text{ at } \cos \pi t = -1 \Rightarrow v_{\min} = b \pi$$

$$\theta = \pi t = \pi, 3\pi, 5\pi$$

$$t = 1, 3, 5, \dots$$

$$v_{\max} \text{ at } \cos \pi t = 1 \Rightarrow v_{\max} = 3b\pi$$

$$\theta = \pi t = 0, 2\pi, 4\pi$$

$$t = 0, 2, 4, 6, \dots$$

$$\cos^2 \pi t \quad \leftarrow \begin{matrix} \text{لوسيني} \\ \min = 0 \quad \max = 1 \end{matrix}$$

$$\sqrt{5 - 4 \cos \pi t} \quad \leftarrow \begin{matrix} \min = 1 \quad \max = -1 \end{matrix}$$



Note  $\vec{v} \cdot \vec{a} = v_R a_R + v_\theta a_\theta + v_z a_z$

$$\vec{v} \times \vec{a} = \begin{vmatrix} \hat{r} & \hat{\theta} & \hat{k} \\ v_R & v_\theta & v_z \\ a_R & a_\theta & a_z \end{vmatrix}$$

problem (11-181) page 679

$$R = A \quad \theta = 2\pi t \quad z = \frac{A t^2}{4}$$

calculate  $\vec{v}, \vec{a}$   
& the time at which  $\vec{v} \perp \vec{a}$

$$\begin{aligned} \dot{R} &= 0 & \dot{\theta} &= 2\pi & \dot{z} &= \frac{At}{2} \\ \ddot{R} &= 0 & \ddot{\theta} &= 0 & \ddot{z} &= \frac{A}{2} \end{aligned}$$

$$\begin{aligned} \vec{v} &= (\dot{R})\hat{r} + (R\dot{\theta})\hat{\theta} + \dot{z}\hat{k} \\ &= (2\pi A)\hat{\theta} + \left(\frac{At}{2}\right)\hat{k} \end{aligned}$$

$$\begin{aligned} \vec{a} &= (\ddot{R} - R\dot{\theta}^2)\hat{r} + (2\dot{R}\dot{\theta} + R\ddot{\theta})\hat{\theta} + \ddot{z}\hat{k} \\ &= (-4\pi^2 A)\hat{r} + \left(\frac{A}{2}\right)\hat{k} \end{aligned}$$

$$\vec{v} \perp \vec{a} \quad \text{when} \quad \vec{v} \cdot \vec{a} = 0$$

$$\begin{aligned} v_R a_R + v_\theta a_\theta + v_z a_z &= 0 \\ (0)(-4\pi^2 A) + (2\pi A)(0) + \left(\frac{At}{2}\right)\left(\frac{A}{2}\right) &= 0 \end{aligned}$$

$$\frac{A^2}{4} t = 0 \quad \Rightarrow \quad t = 0$$

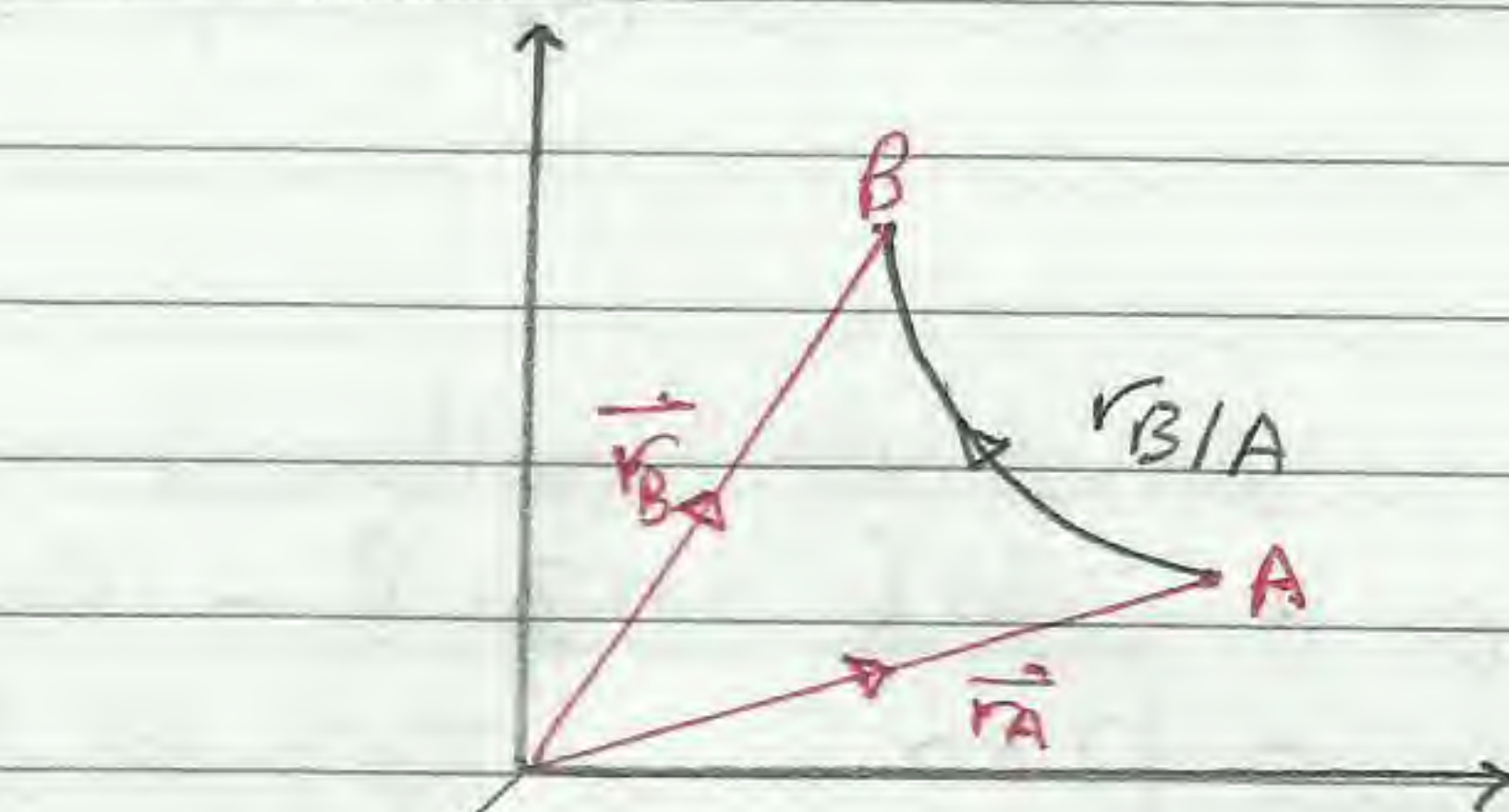
A can't be 0

$$\vec{v} \parallel \vec{a} \quad \text{if} \quad \vec{v} \text{ unit} \parallel \vec{a} \text{ is a straight line}$$

تكون في اتجاه واحد



# Relative motion



$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$$

از B بالنسبه الى A

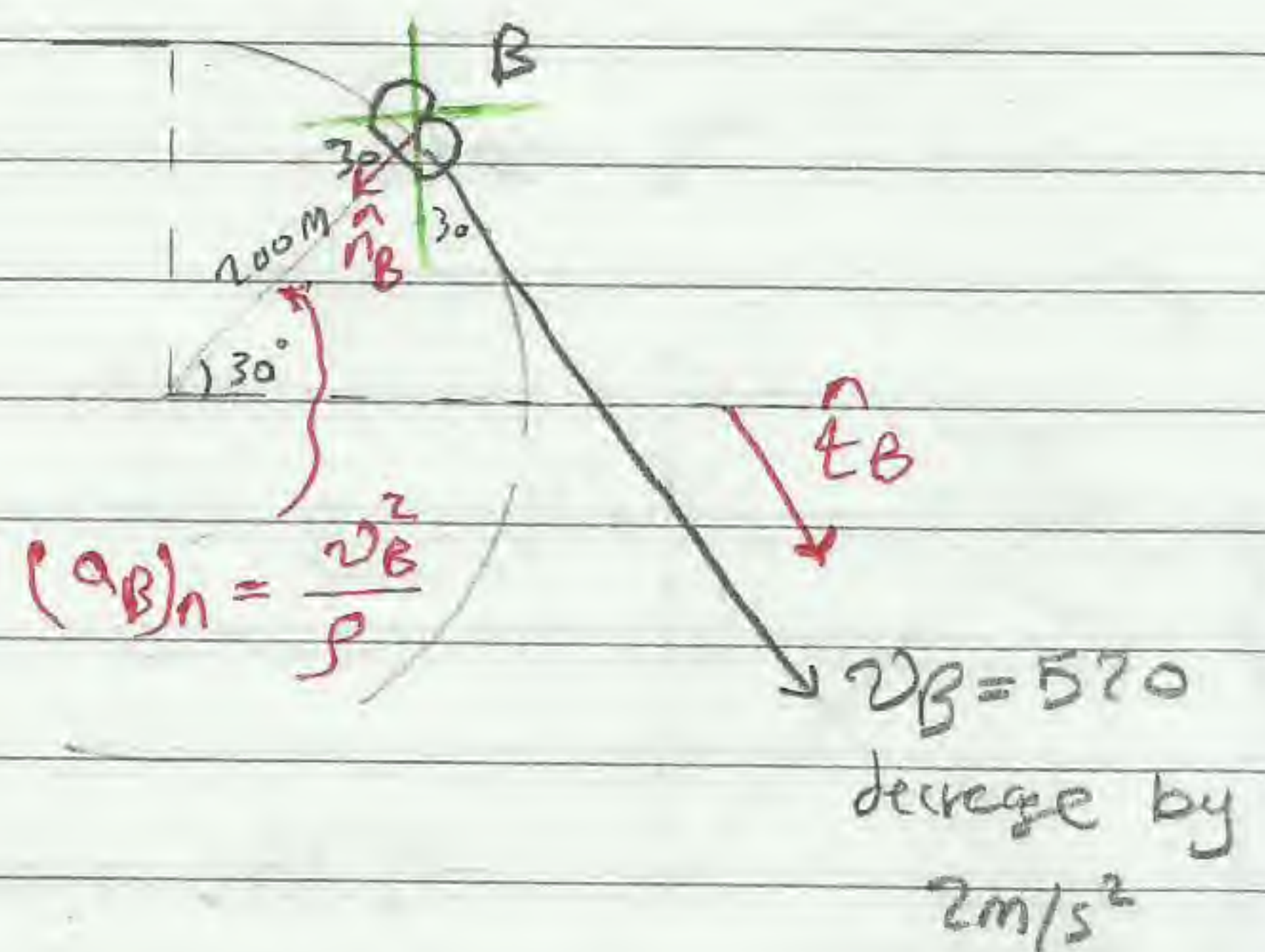
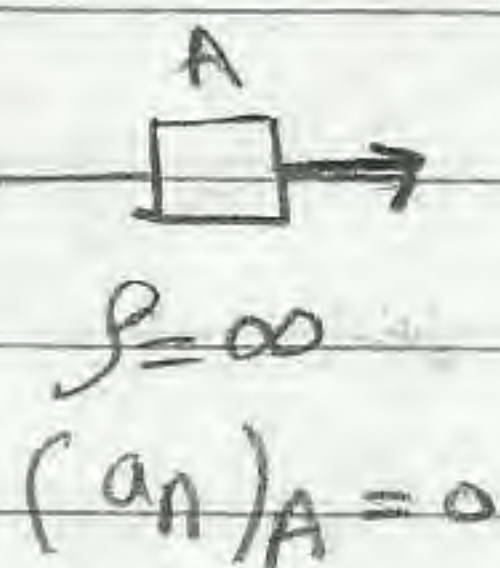
$$\begin{aligned} \vec{r}_{B/A} &= \vec{r}_B - \vec{r}_A \\ \vec{v}_{B/A} &= \vec{v}_B - \vec{v}_A \\ \vec{a}_{B/A} &= \vec{a}_B - \vec{a}_A \end{aligned}$$

$$\vec{v} = v \hat{t} = \dot{s} \hat{t}$$

$$\vec{a} = \dot{v} \hat{t} + \left( \frac{v^2}{\rho} \right) \hat{n}$$

problem (11.143) page 673

$v_A = 420 \text{ km/hr}$  increase by  $6 \text{ m/s}^2 = a_t = a$



calculate  $\vec{v}_{B/A}$   
 $\vec{a}_{B/A}$

مخارج و زوايا

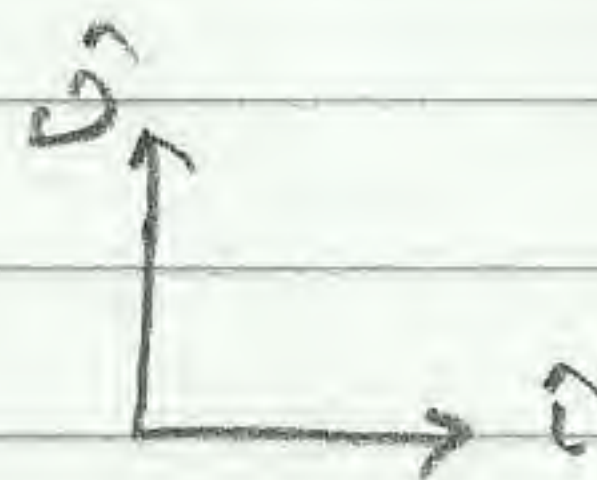
عن امل و زوايا



$$\vec{v}_{B/A} = \vec{v}_B - \vec{v}_A$$

$$\vec{a}_{B/A} = \vec{a}_B - \vec{a}_A$$

$$\vec{v}_A = 420 \hat{i}$$



$$\begin{aligned} \vec{v}_B &= 520 \hat{t}_B = 520 (\sin 30^\circ \hat{i} - \cos 30^\circ \hat{j}) \\ &= 260 \hat{i} - 260\sqrt{3} \hat{j} \end{aligned}$$

$$\vec{v}_{B/A} = -160 \hat{i} - 260\sqrt{3} \hat{j} \quad \text{Km/hr}$$

$$\vec{a}_A = 6 \hat{i}$$

$$\vec{a}_B = -2 \hat{t}_B + \frac{(520 \times \frac{5}{18})^2}{200} \hat{n}_B$$

$$= -2(\sin 30^\circ \hat{i} - \cos 30^\circ \hat{j}) + \frac{(520 \times \frac{5}{18})^2}{200} (-\cos 30^\circ \hat{i} - \sin 30^\circ \hat{j})$$

$$\begin{aligned} 418 \hat{i} \\ 418 \hat{j} \end{aligned}$$



## Kinetics of a particle

Relates the properties of motion & its reasons (force)

force - Acceleration principle. قانون نيوتن الثاني

$$\vec{F} = m \vec{a}$$

Cartesian

$$\sum F_x = m \ddot{x}$$

$$\sum F_y = m \ddot{y}$$

polar

$$\sum F_R = m (\ddot{R} - \dot{\theta}^2 R)$$

$$\sum F_\theta = m (2\dot{R}\dot{\theta} + R\ddot{\theta})$$

Intrinsic

$$\sum F_t = m \dot{v}$$

$$\sum F_n = m \left( \frac{v^2}{\rho} \right)$$

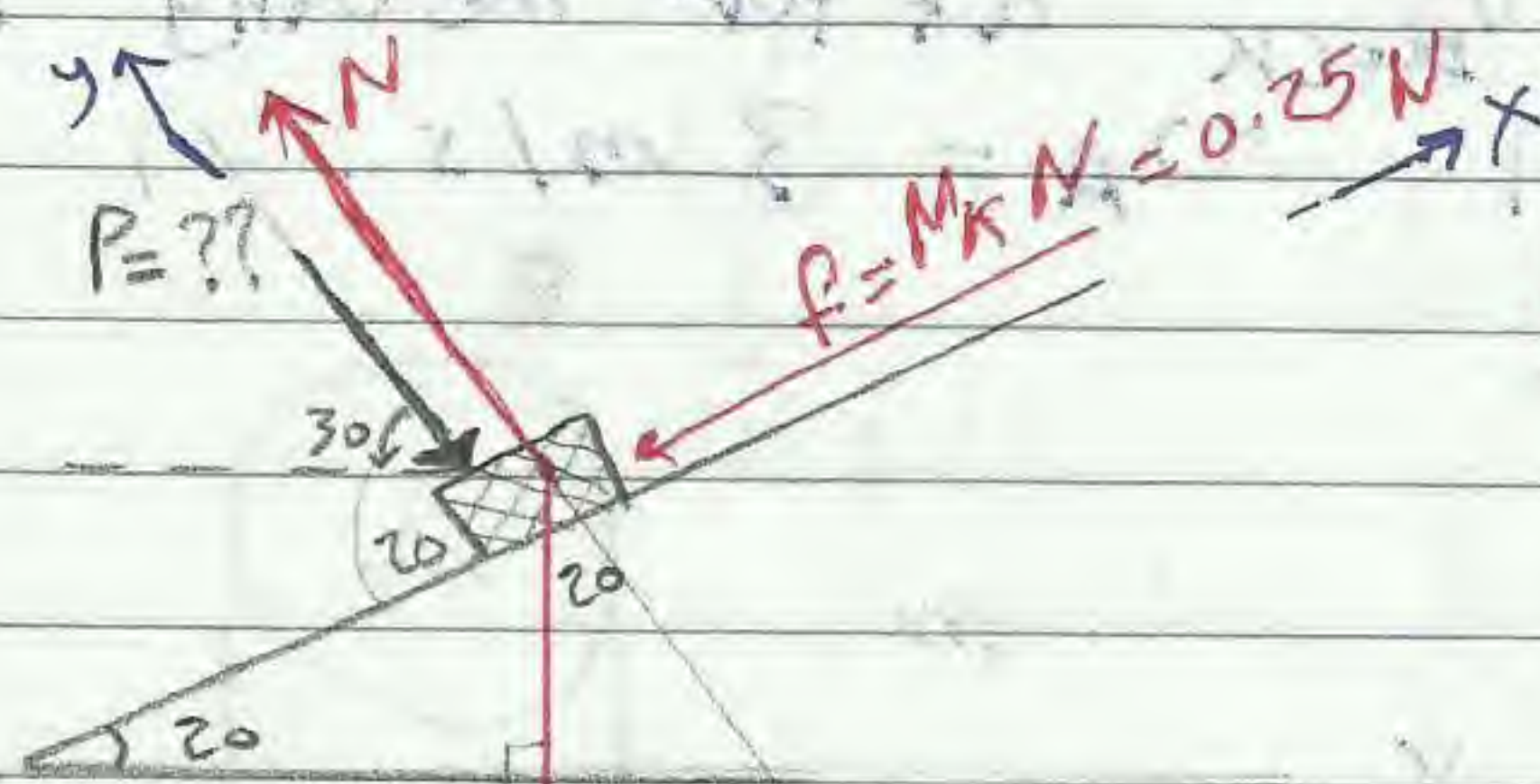
problem (12-10) page 707

$$m = 40 \text{ Kg} \quad v_0 = 0$$

$$\mu_s = 0.3$$

$$\mu_k = 0.25$$

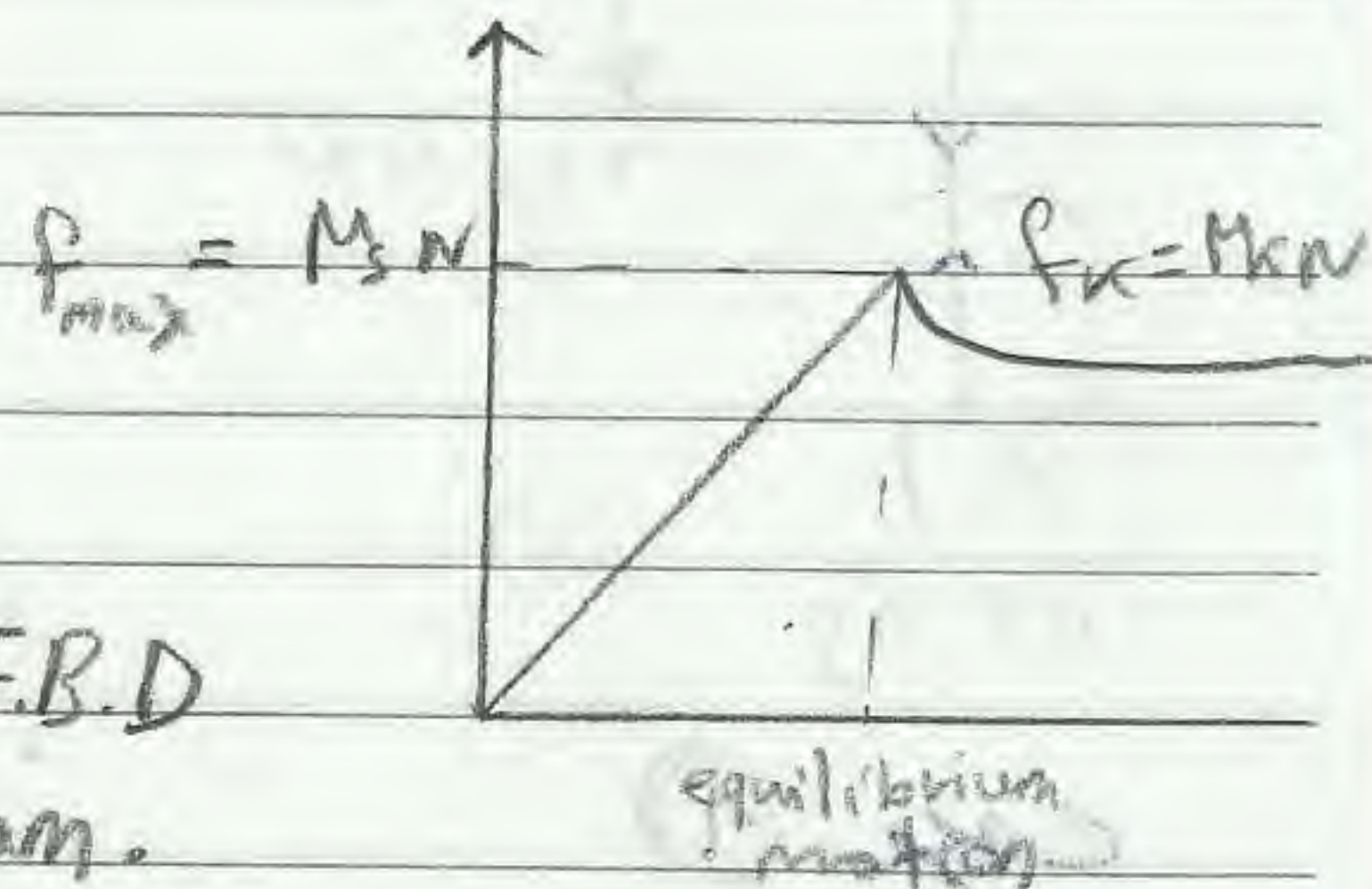
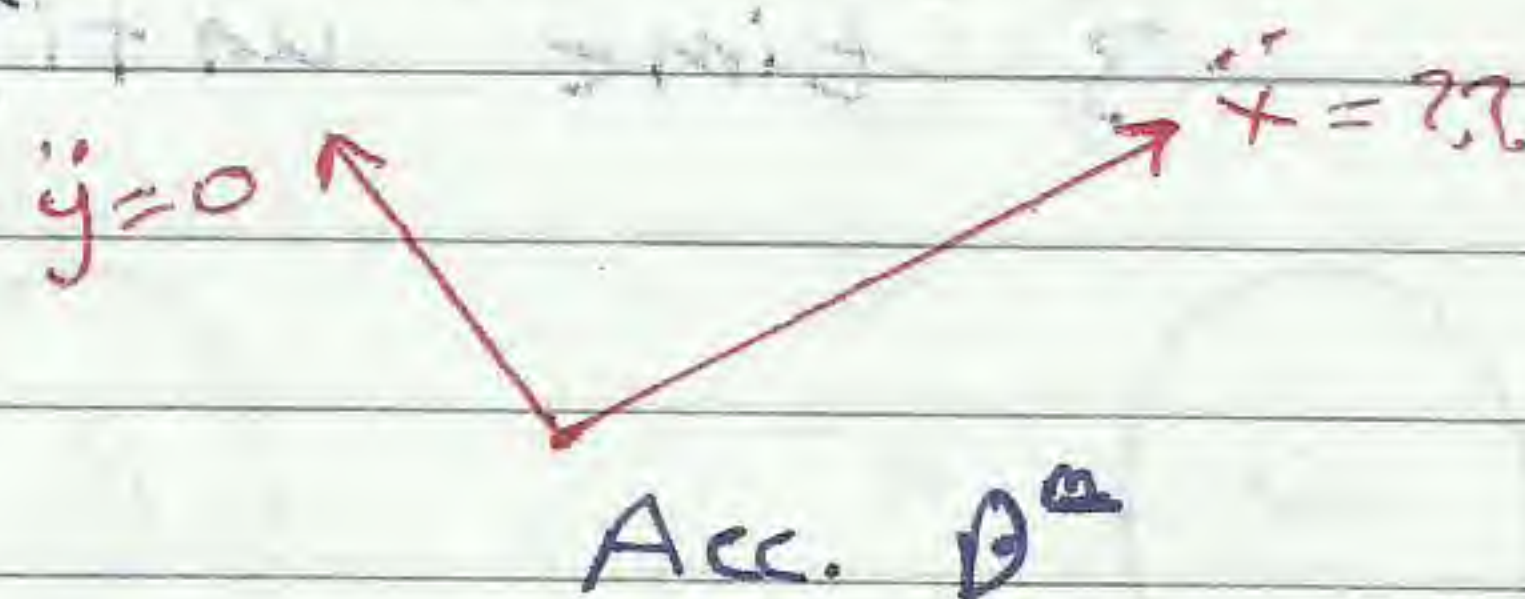
calculate  $P = ??$  such that the block moves to m into 4s.



$$W = 40(9.8) = 392 \text{ Newton}$$

Free body diagram

plane parallel to the incline  $\ddot{y} = 0$



steps of solutions:-

1 Draw the Free body diagram F.B.D

2 Draw the Acceleration & Diagram.

3 write the equation of motion.

4 solve for unknowns.



equation of motion

$$\sum F_x = m \ddot{x} \Rightarrow P \cos 50^\circ + 392 \sin 20^\circ - 0.25 N = 40 a \quad (1)$$

$$\sum F_y = m \ddot{y} \Rightarrow N - P \sin 50^\circ - 392 \cos 20^\circ = 40 a \quad (2)$$

a constant  $\leftarrow v_{\text{constant}}$   
a variable  $\leftarrow v_{\text{variable}}$

since the forces are constant

$\Rightarrow a = \text{constant}$  (Rectilinear)

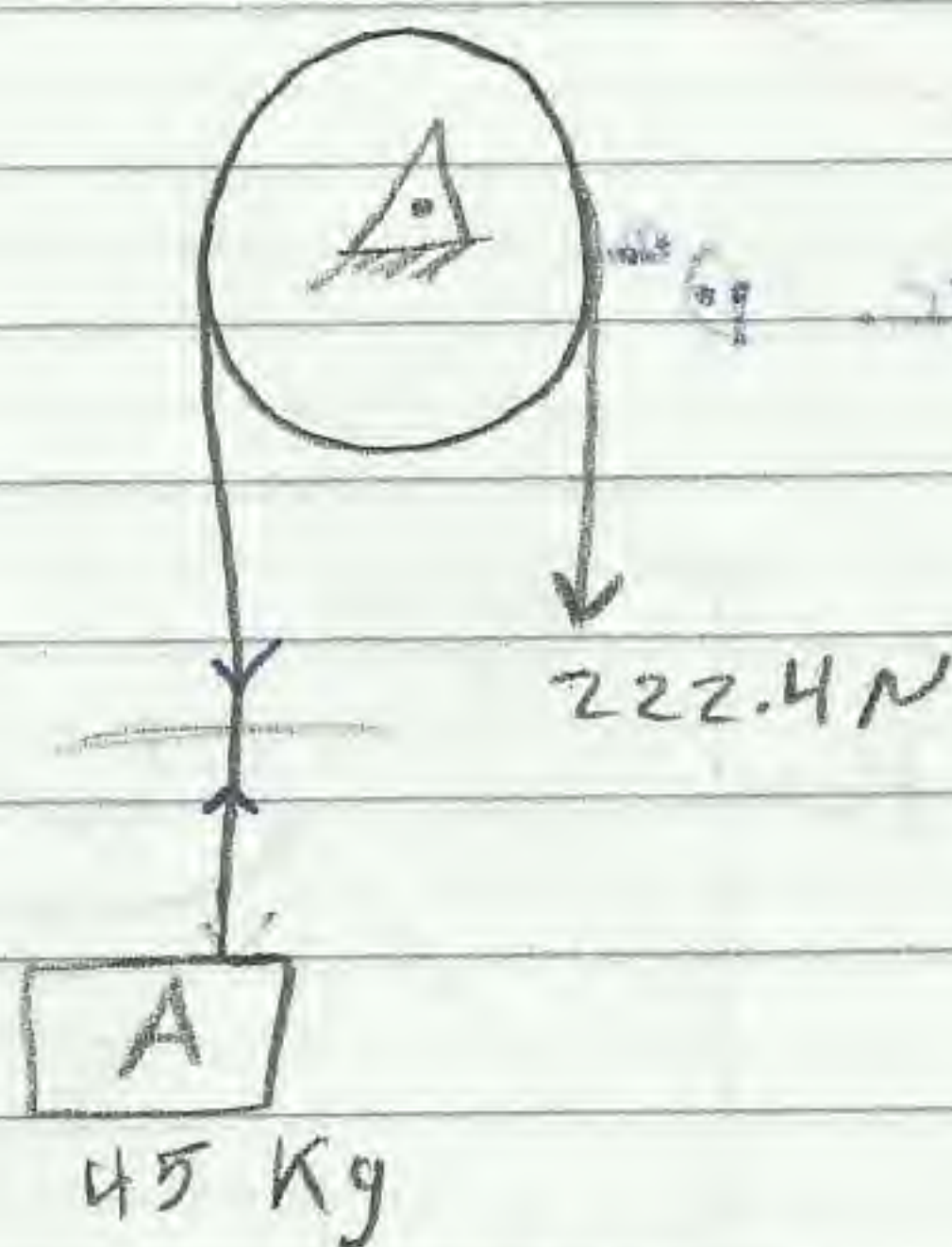
$$d = v_0 t + \frac{1}{2} a t^2$$

$$10 = (0)(4) + \frac{1}{2} a (4)^2 \Rightarrow a = 1.25 \text{ m/s}^2$$

from (1), (2) get  $P, N$

Problem (12.20) page 708

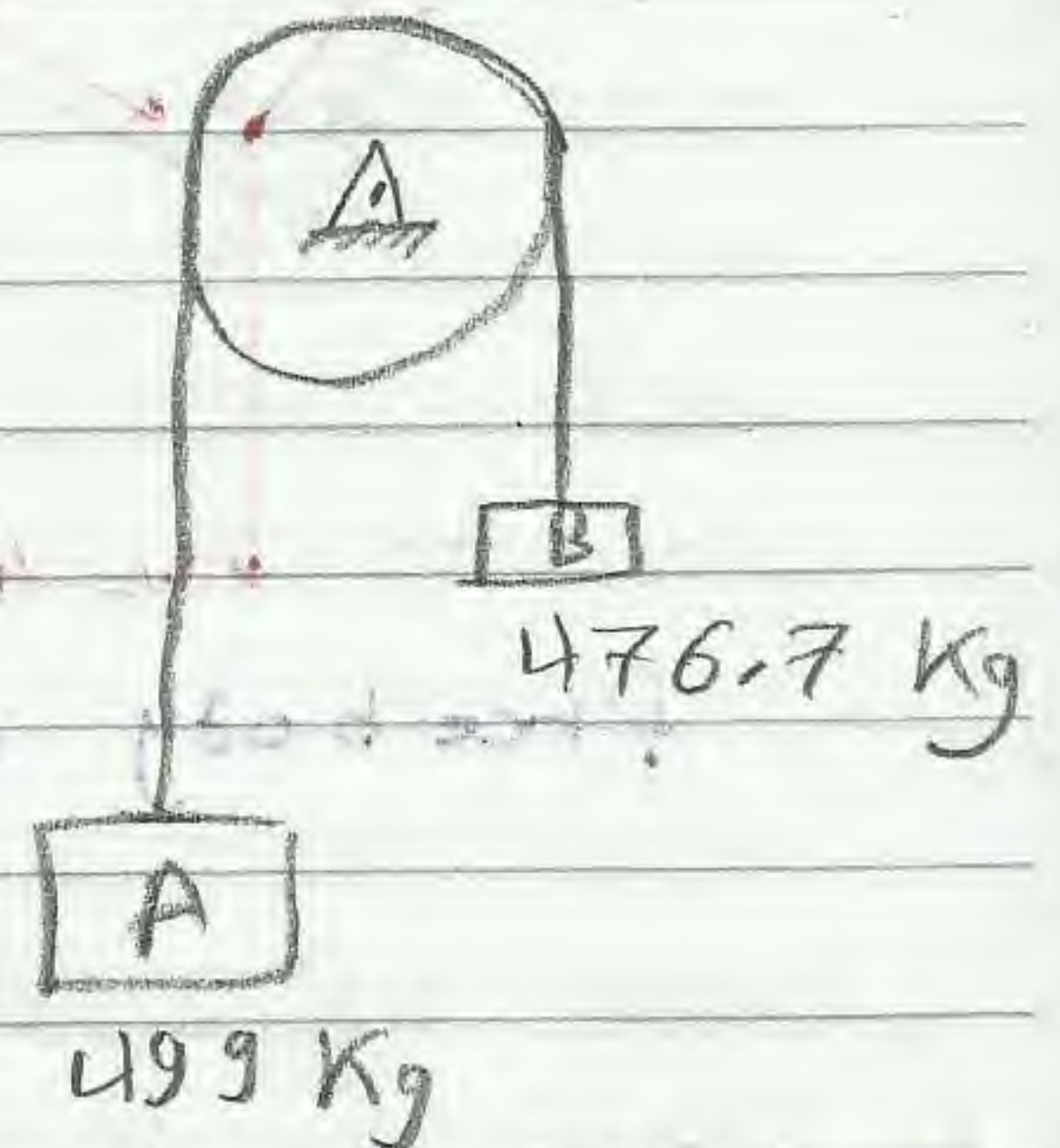
calculate (1)  $a_{\text{block A}}$ , (2)  $v_{\text{block A}}$  after moving 1.5 m  
(3) time until  $v_A = 3 \text{ m/s}$



Case (1)



Case (2)

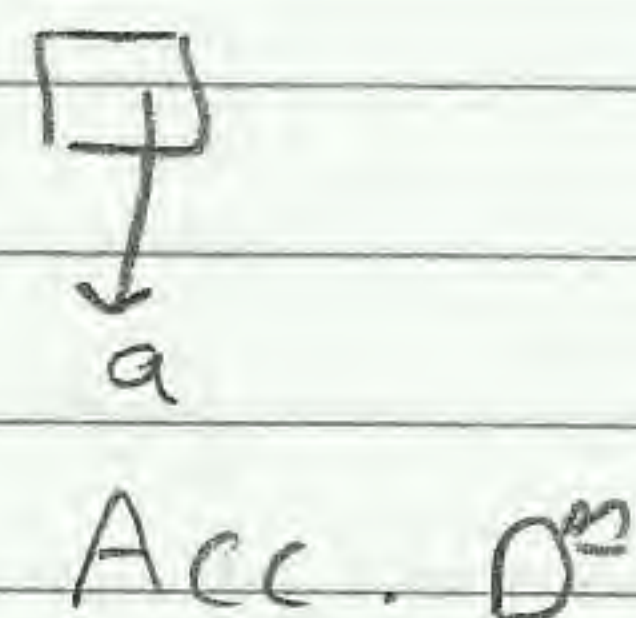
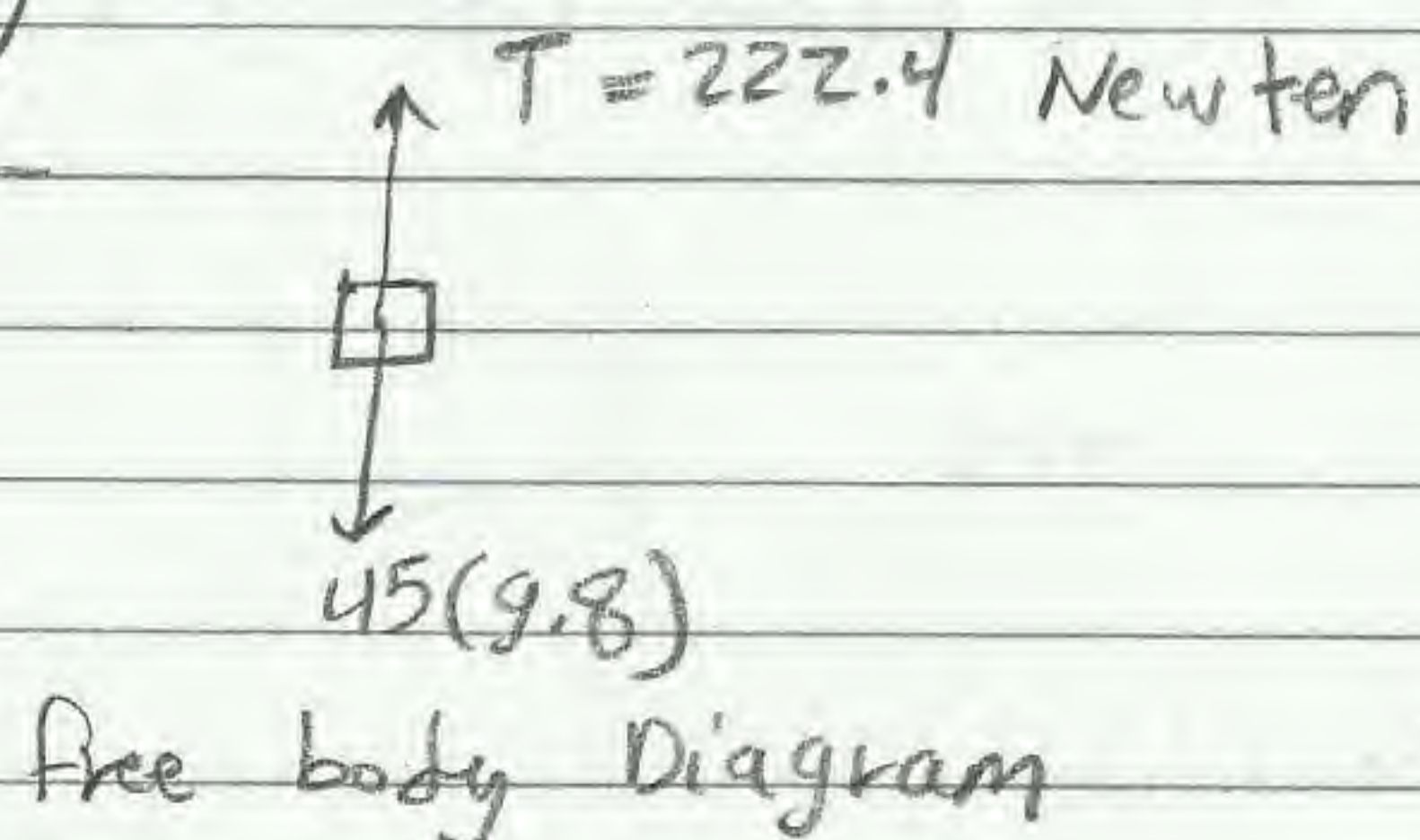


Case (3)

مكرر



Case ①



Equation of motion:

$$45(9.8) - 222.4 = 45a \Rightarrow a = \dots \text{ m/s}^2$$

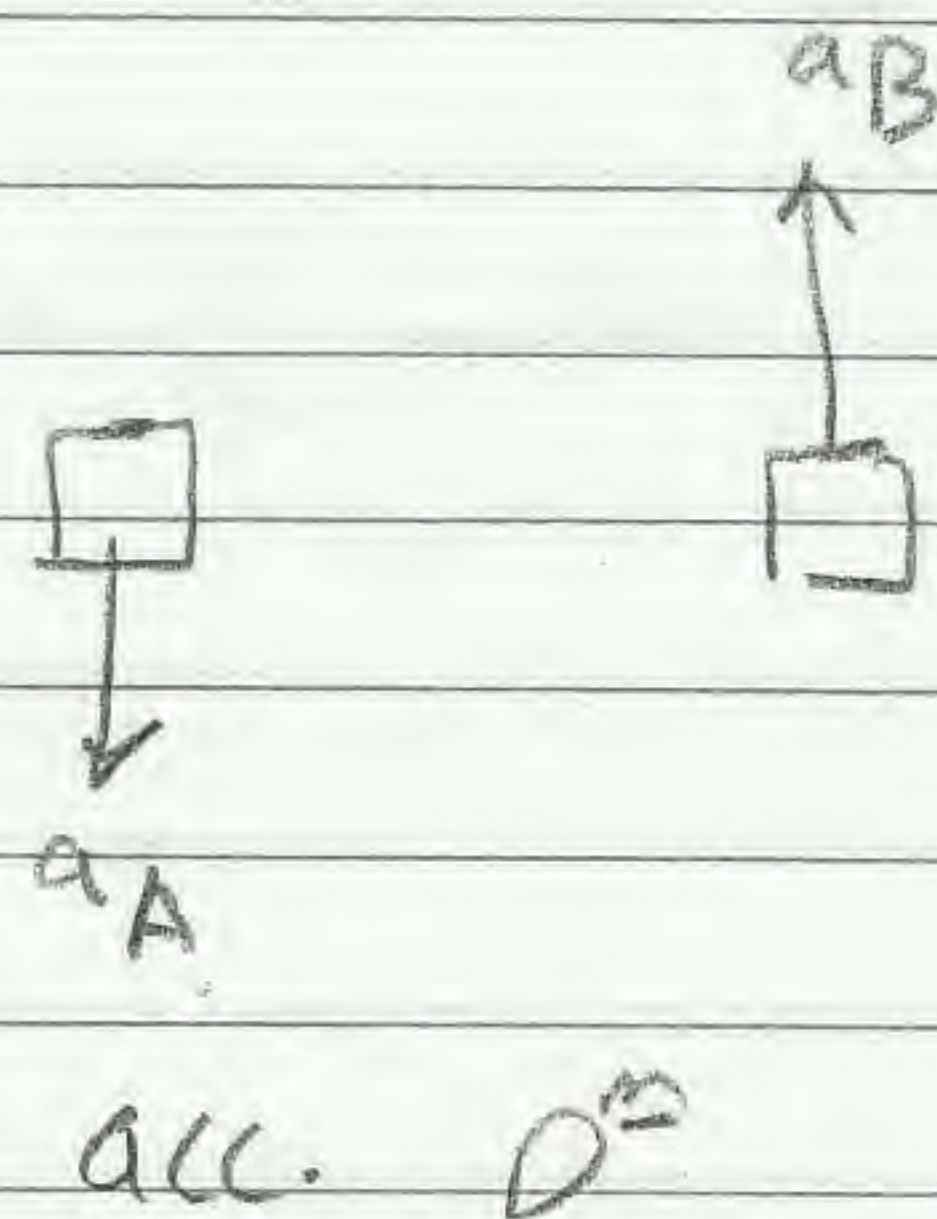
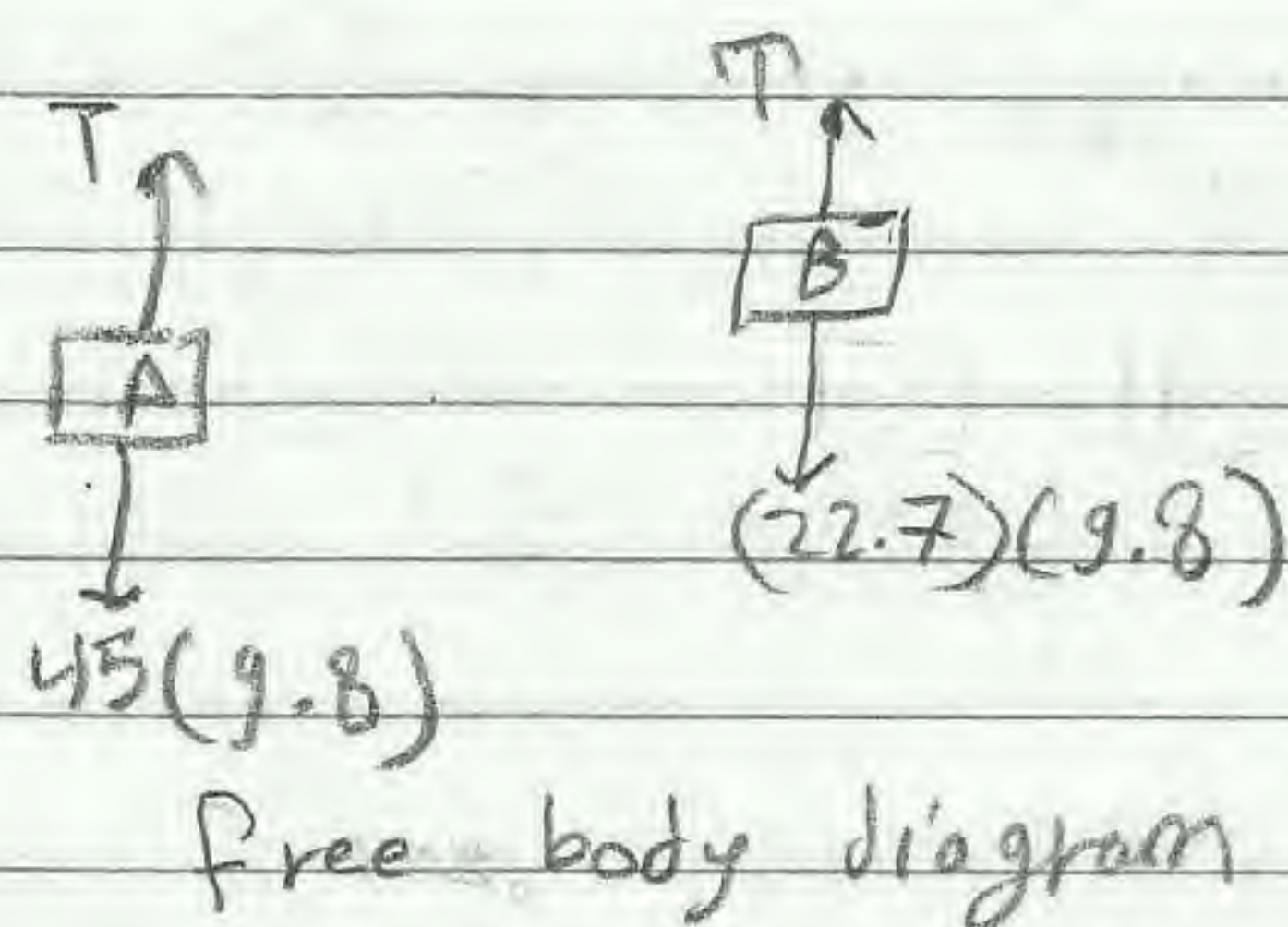
$$v^2 = v_0^2 + 2ad = 0^2 + 2(\dots)(1.5)$$

$$v_A = \dots \text{ m/s}$$

$$v = v_0 + at \Rightarrow 3 = 0 + (\dots)t$$

$$t = \checkmark$$

Case ②



Equation of motion

A  $(45)(9.8) - T = 45a_A$  ———— (1)

B  $T - (22.7)(9.8) = 22.7a_B$  ———— (2)

$a_A = a_B$

$$m_A g - m_B g = (m_A + m_B) a_A$$

$$45(9.8) - (22.7)(9.8) = (45 + 22.7) a_A$$



problem (12.46) page 713

use a good line ↓



The speed decrease with a constant rate.  
Calculate the change in forces on a passenger  
 $m = 90.7$  at B.

int.

$$\sum F_t = m \dot{v} \quad , \quad \sum F_n = m \left( \frac{v^2}{r} \right)$$

solution:

kinematics

$\therefore a_t = \text{constant}$

$$\therefore v_C^2 = v_A^2 + 2 a_t s_{A \rightarrow C}$$

$$(146)^2 = (165)^2 + 2 a_t \left( 6400 \times \frac{8\pi}{180} + 800 \right)$$

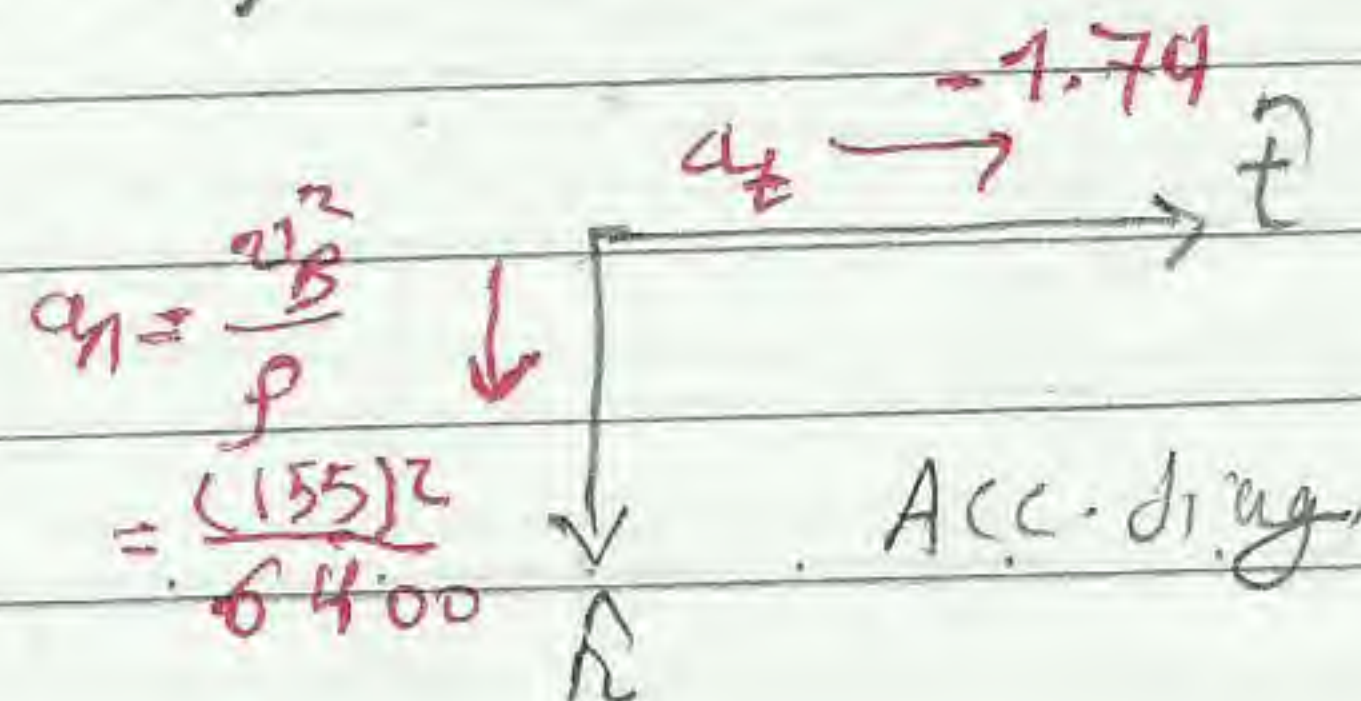
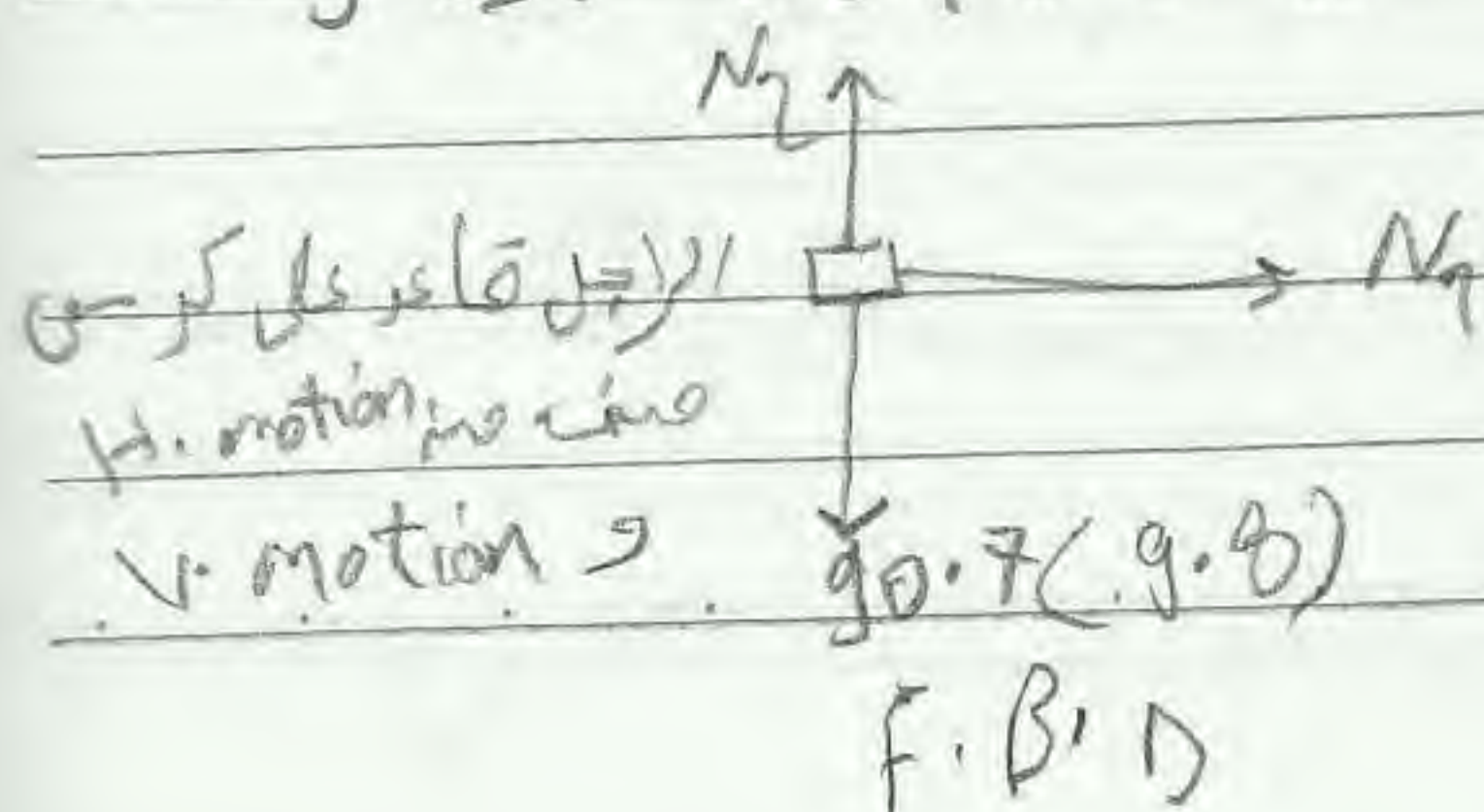
$$a_t = -1.74 \text{ m/s}^2$$

$$v_C^2 = v_B^2 + 2 a_t s_{B \rightarrow C}$$

$$(146)^2 = v_B^2 + 2 (-1.74) (800)$$

$$v_B = 155 \text{ m/s}$$

Just before B (curve)





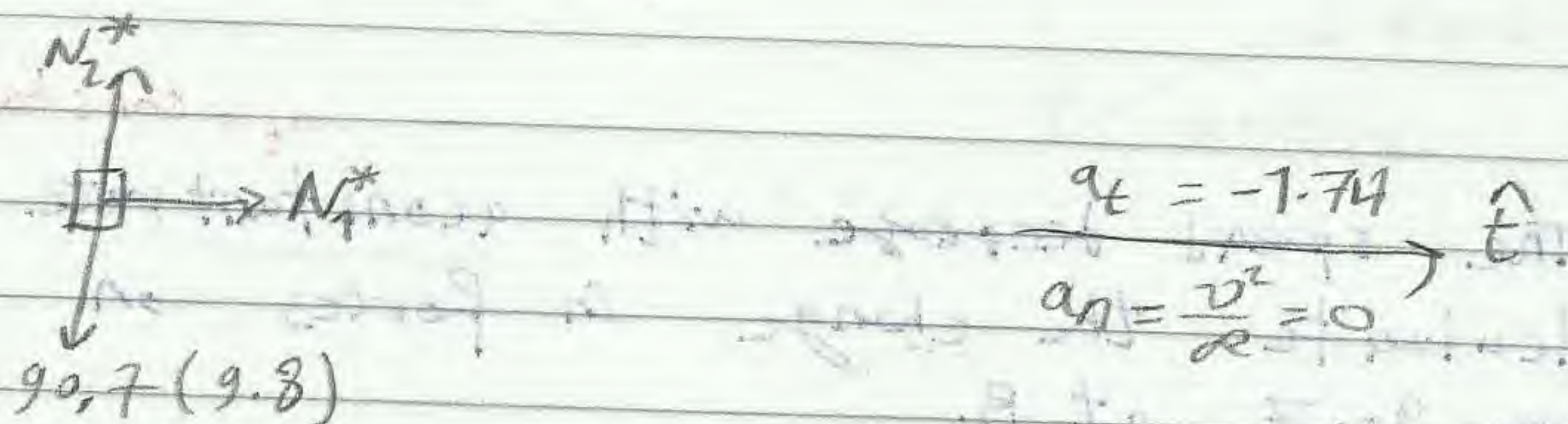
$\hat{t} \parallel$

$$N_1 = 90.7(-1.74)$$

$\hat{n} \perp$

$$(90.7)(9.8) - N_2 = (90.7) \left( \frac{(155)^2}{6400} \right)$$

Just after B (line)



$\hat{t} \parallel$

$$N_1^* = (90.7)(-1.74)$$

$$(90.7)(9.8) - N_2^* = m(0)$$

$$\Delta N_2 = N_2^* - N_2 = m \frac{v_B^2}{r} = \checkmark$$

Normal زارت

واحد

50 Kilo

واحد

70 Kilo

↑ دة بعث بار change { كس

←

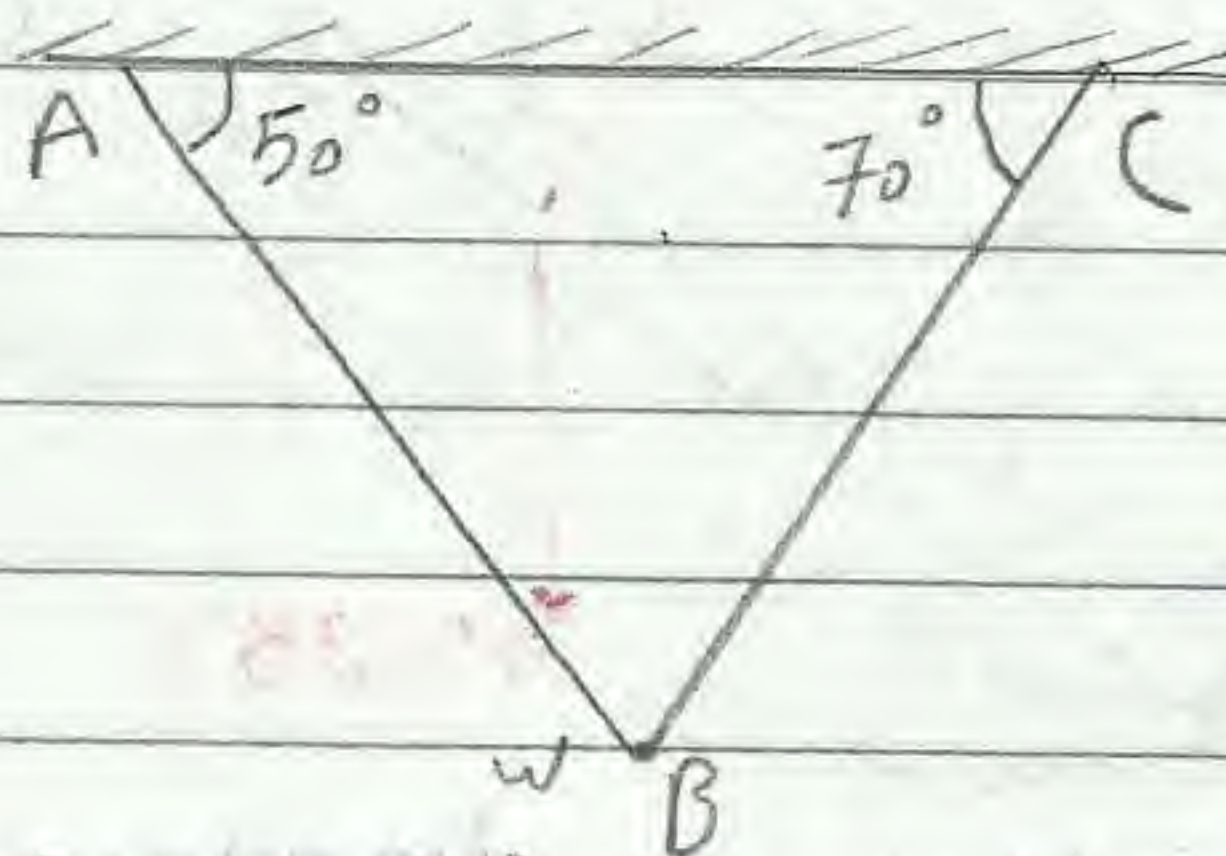
↪ لو ال speed زارت ← يزيه التاشين

$$\Delta N_2 \propto \frac{1}{r}$$

↪ لو r زارت ← التاشين يقل



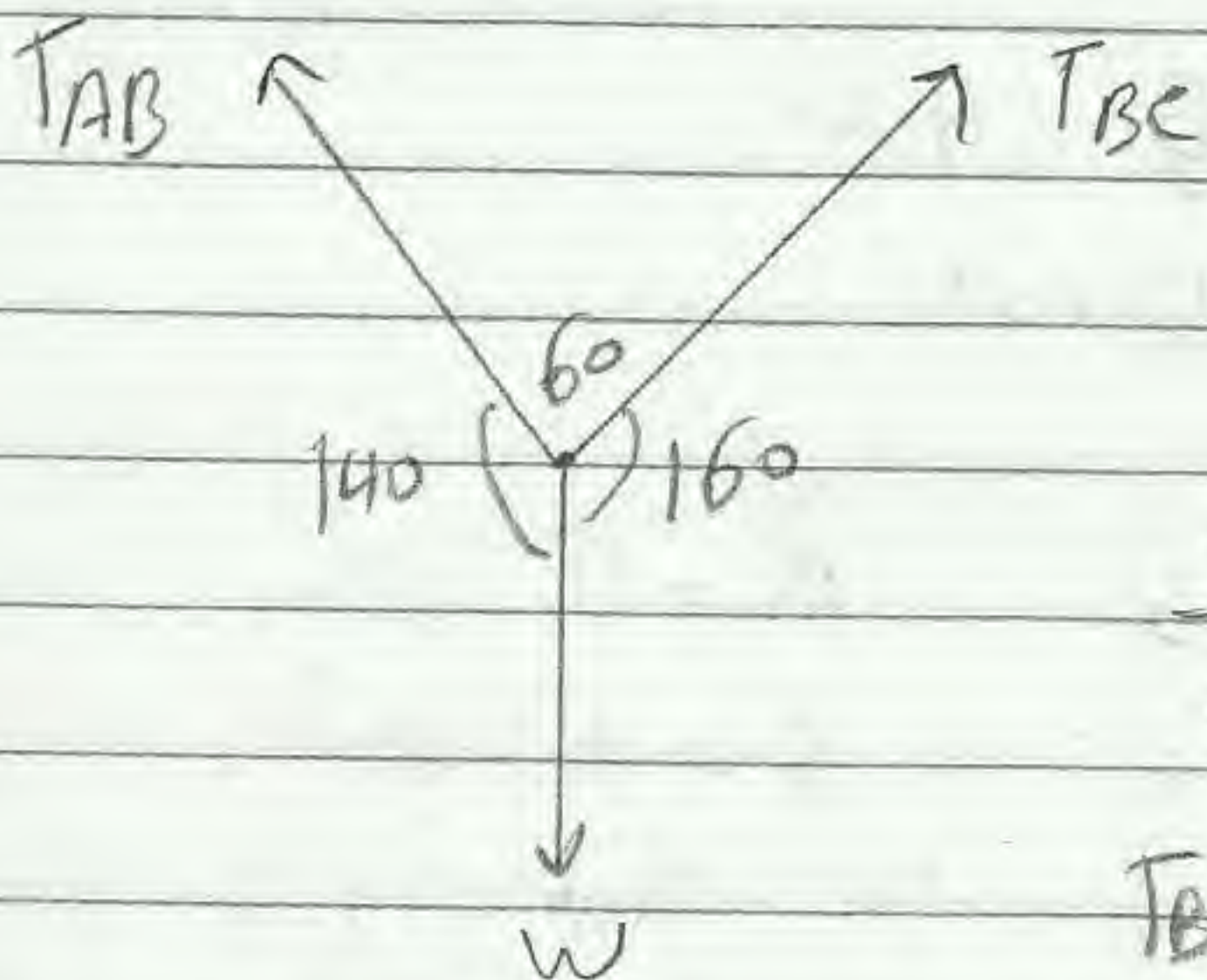
problem (12-44) page 713 (82.87) problems



calculate the tension in cable BC

i) Before AB is cut.

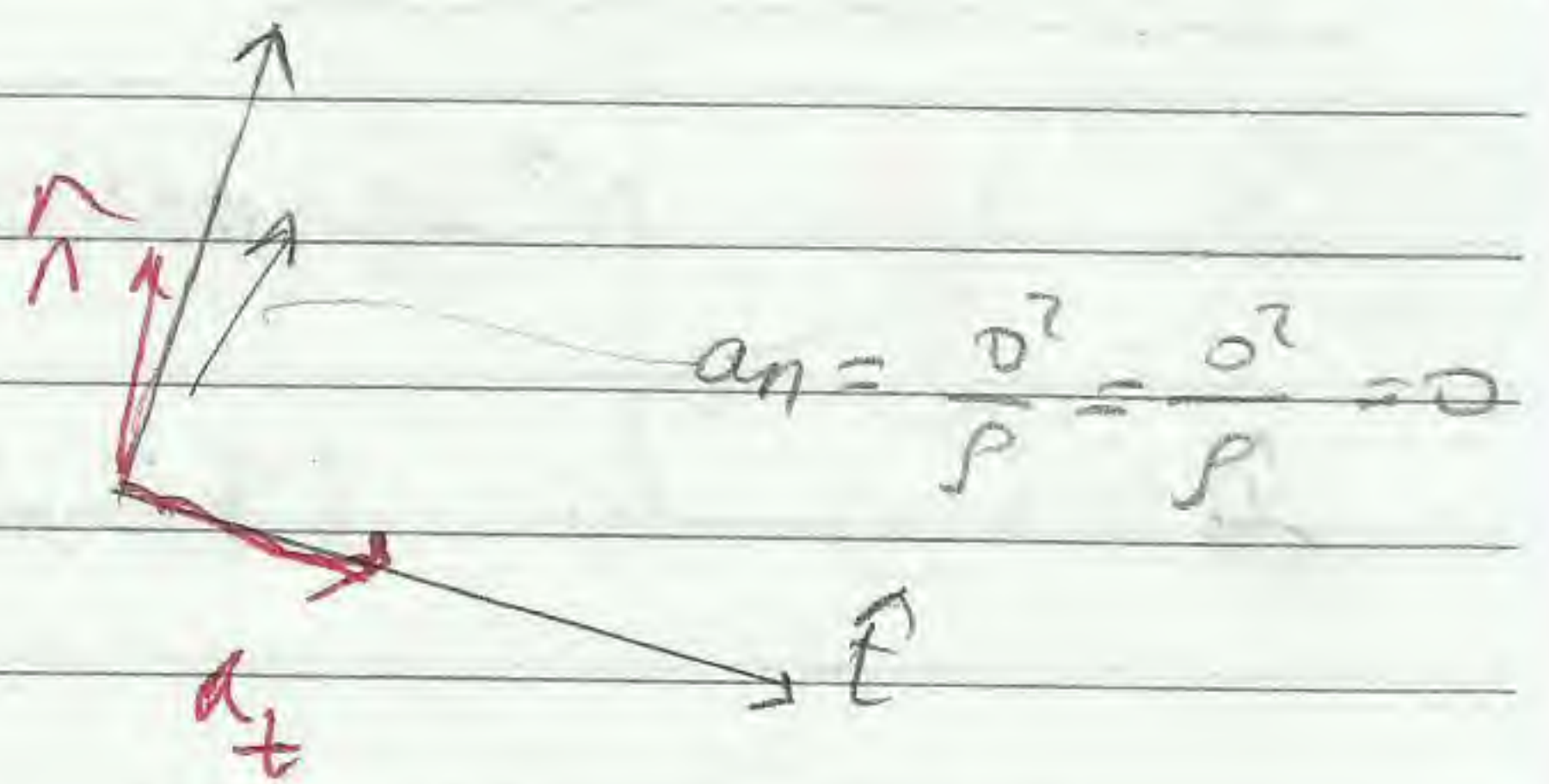
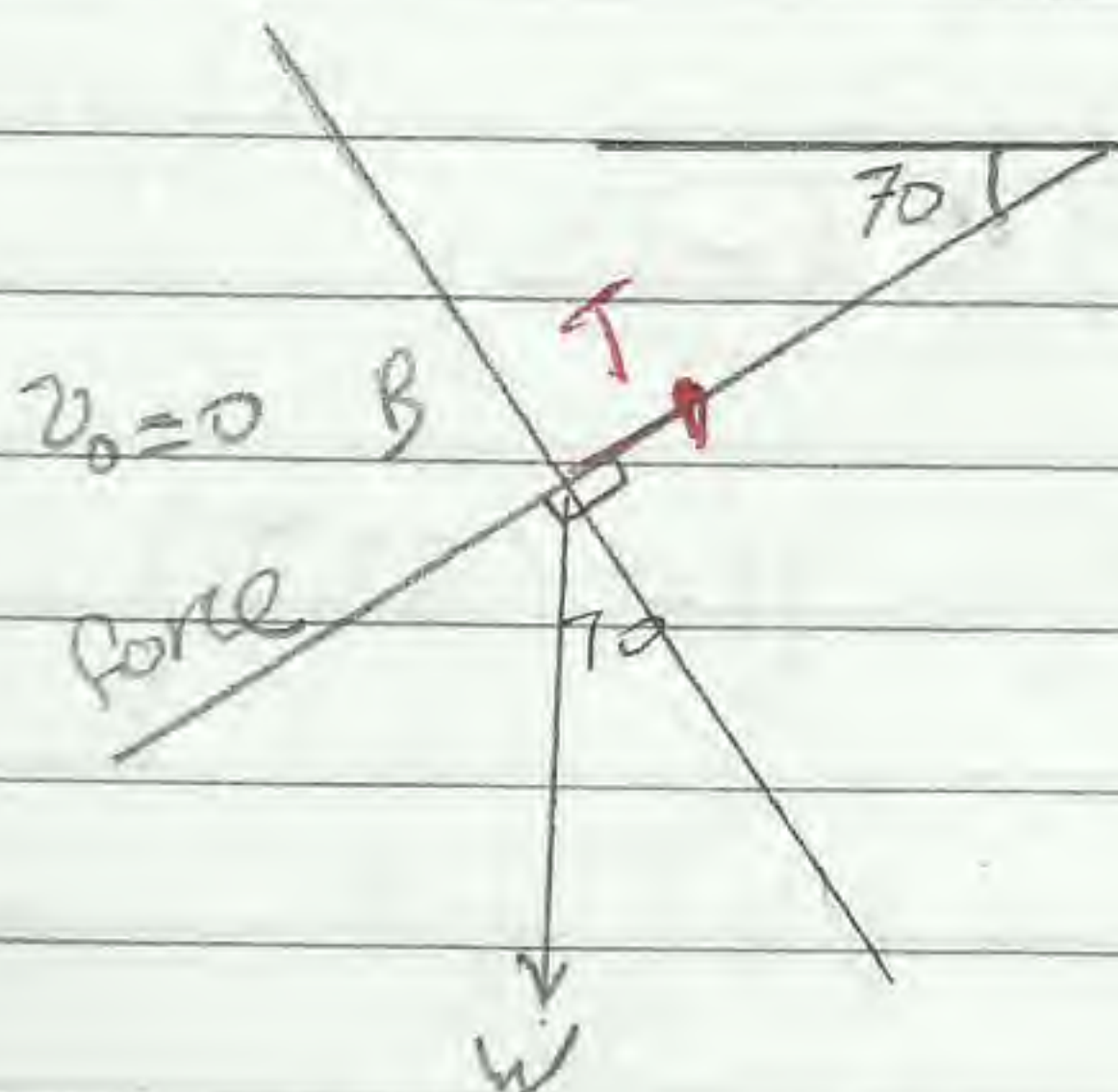
ii) Just after AB is cut.



$$\frac{w}{\sin 60} = \frac{T_{BC}}{\sin 140} = \frac{T_{AB}}{\sin 160}$$

$$T_{BC} = w \left( \frac{\sin 140}{\sin 60} \right)$$

Just after AB is cut



$$a_n = \frac{v^2}{\rho} = \frac{0^2}{\rho} = 0$$

$$w \cos 70 = m a_t$$

$$a_t = g \cos 70$$

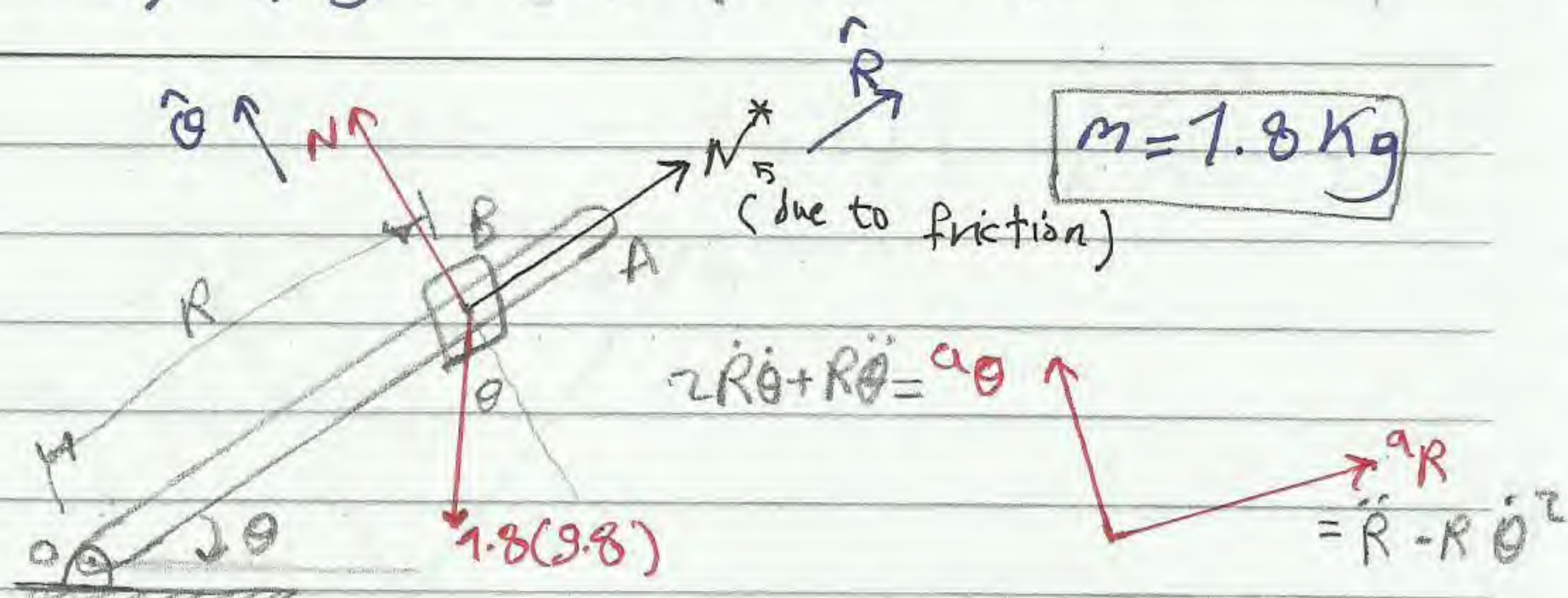
$$T_{BC}^* - w \sin 70 = m \left( \frac{0^2}{\rho} \right)$$

$$T_{BC}^* = w \sin 70$$

just after particle is released at B



problem (12.68) page 726



calculate the radial & transverse reactions at  $t=0$  &  $t=1\text{s}$ .

Friction force

$$R = 3t^2 - t^3$$

$$\theta = 2t^2$$

$$\dot{R} = 6t - 3t^2$$

$$\dot{\theta} = 4t$$

$$\ddot{R} = 6 - 6t$$

$$\ddot{\theta} = 4$$

at  $t=1\text{s}$ :  $R = 2\text{m}$ ,  $\dot{R} = 3\text{m/s}$ ,  $\ddot{R} = 0$

$\theta = 2\text{rad}$ ,  $\dot{\theta} = 4\text{r/s}$ ,  $\ddot{\theta} = 4\text{r/s}^2$

$$a_R = -32 \text{ m/s}^2$$

$$a_\theta = 24 + 8 = 32 \text{ m/s}^2$$

Equations of motion:-  $\frac{2\pi}{180} \times \frac{180}{2\text{rad}}$

$$\hat{R} \quad N - (1.8)(9.8) \sin \theta = 1.8(-32)$$

$$\hat{\theta} \quad N - (1.8)(9.8) \cos \theta = 1.8(32)$$

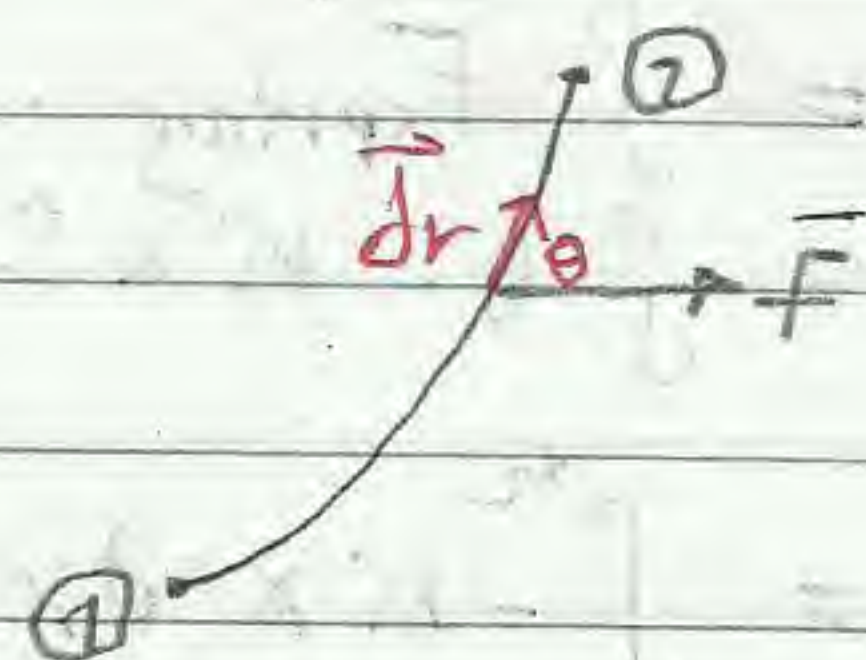


## ✓ chapter (13)

work - Energy (force - ~~speed~~ velocity - distance method)

### ① work

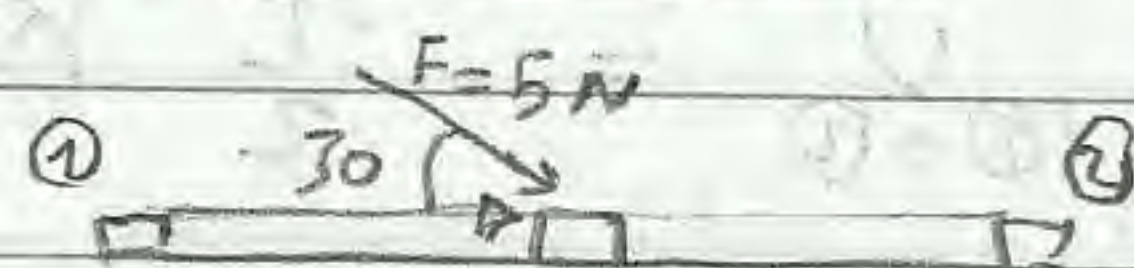
$$U_{1 \rightarrow 2} = \int du = \int \vec{F} \cdot d\vec{r} = \int F(dr) \cos \theta$$



special case:  $F = \text{constant}$ ,  $\theta = \text{constant}$ .

$$U_{1 \rightarrow 2} = F \cdot \text{path length} \cdot \cos \theta$$

↑  
path length



$U_{1 \rightarrow 2}$

+ve

$\theta \in [0, 90[$

force support motion.

zero

$\theta = 90^\circ$



-ve

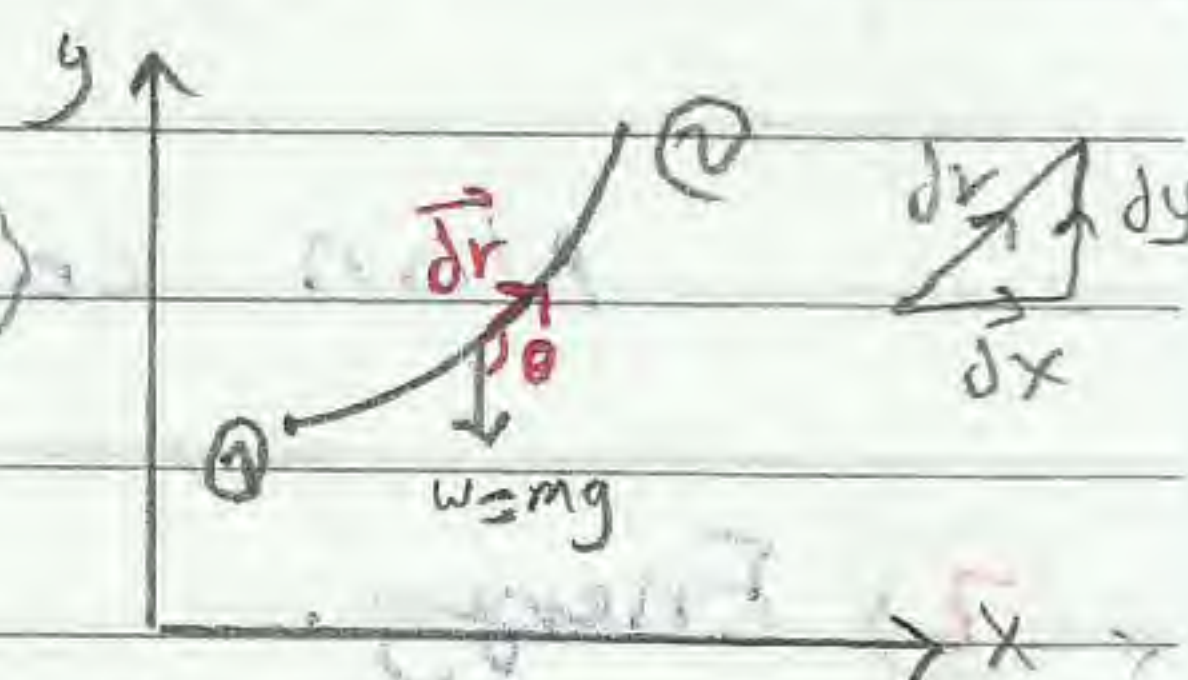
$\theta \in ]90, 180[$

force resist motion

### • Work of common forces:

#### 1. weight

work is independent on path



$$U_{1 \rightarrow 2} = \int \vec{w} \cdot d\vec{r} = \int -mg \hat{j} \cdot (dx \hat{i} + dy \hat{j})$$

$$= -mg \int_{y_1}^{y_2} dy$$

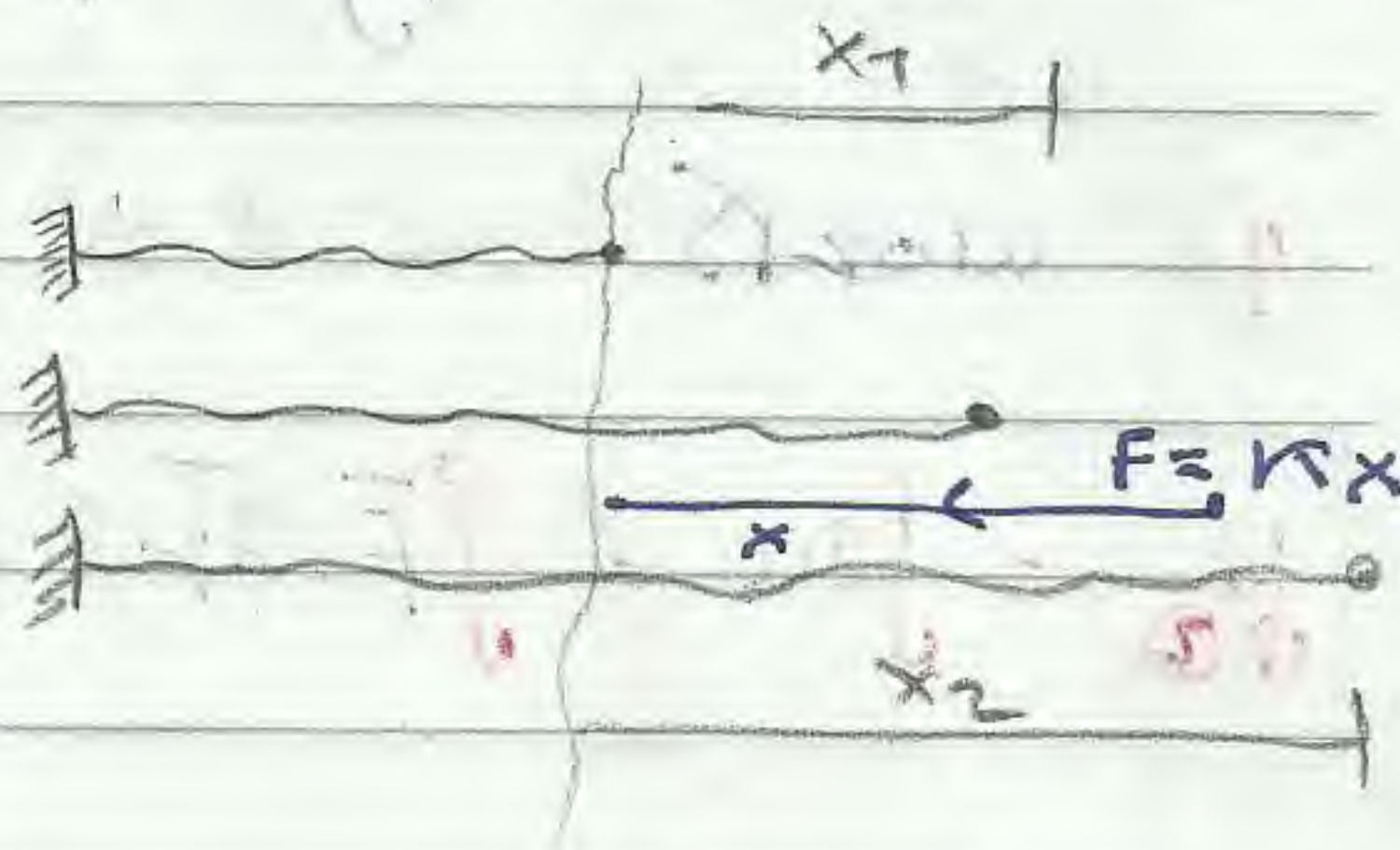
$$U_{1 \rightarrow 2} = mgy_1 - mgy_2 = mg \Delta y$$

لو نازل مع الجاذبية +y لو طالع ↑ يعني -y



## 2. Spring.

$$\begin{aligned}
 U_{Q \rightarrow Q} &= \int \vec{F}_{\text{spring}} \cdot d\vec{r} \\
 &= \int (-Kx \hat{i}) \cdot (dx \hat{i} + dy \hat{j}) \\
 &= \int_{x_1}^{x_2} -Kx dx = -K \frac{x^2}{2} \Big|_{x_1}^{x_2}
 \end{aligned}$$



$$U_{Q \rightarrow Q} = \frac{1}{2} K x_1^2 - \frac{1}{2} K x_2^2$$

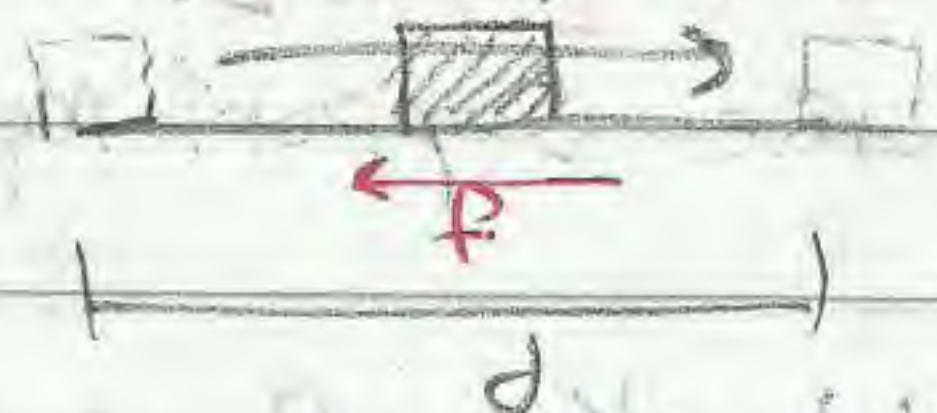
+ve يبقى  $x_2 < x_1$  ←

## 3. Friction

work is dependent on path length

$$U_{Q \rightarrow Q} = f d \cos 180 = -f d$$

ال Fric عكس اتجاه الحركة.

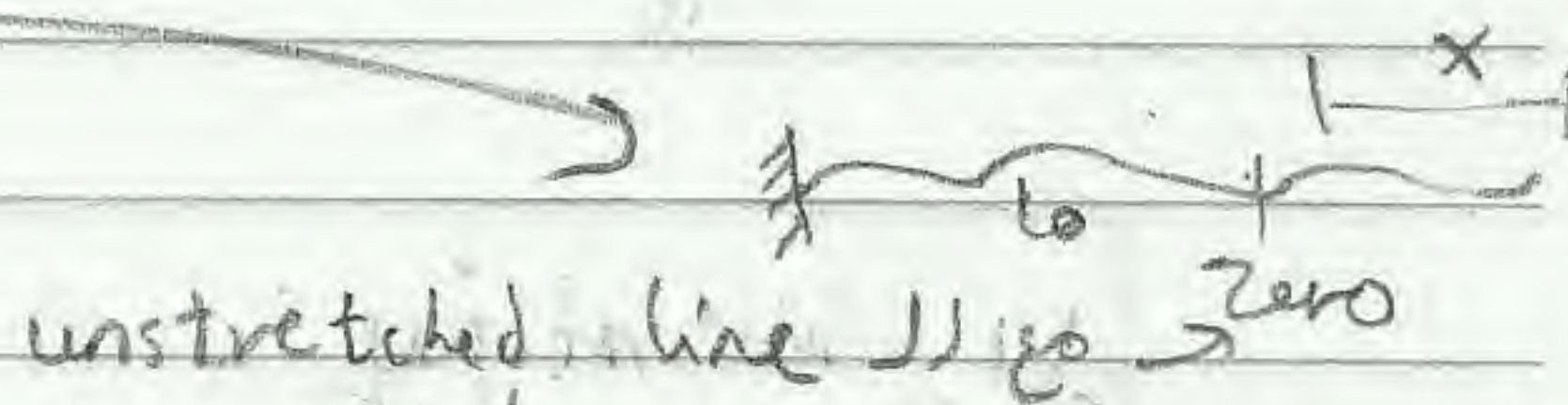
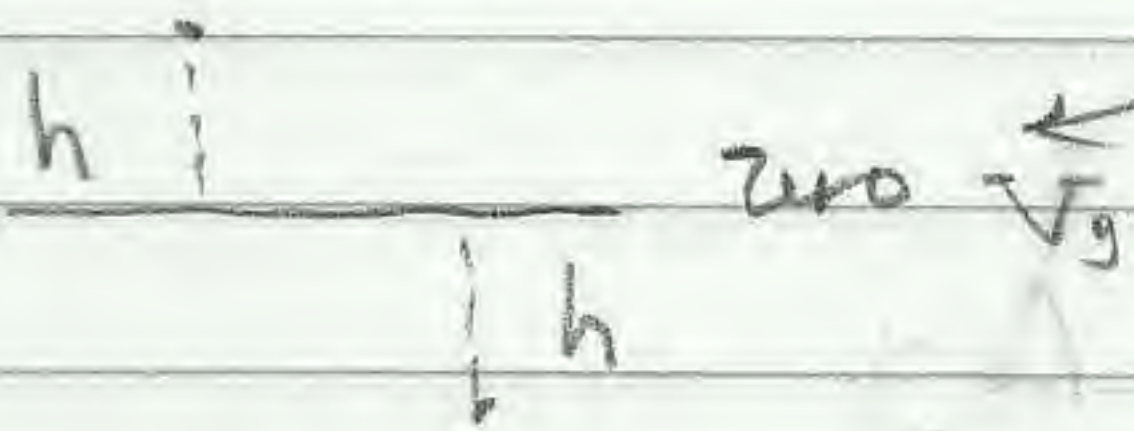


Non conservative.

## ② Energy.

- i) kinetic energy (T)
- ii) potential energy (V)

$$T = \frac{1}{2} m v^2$$



$$V_g = \pm m g h$$

gravity

$$V_s = \frac{1}{2} K x^2$$

spring

+ve (←)



### ③ work - Energy relation. (2.51) x3

$$\boxed{U_{1 \rightarrow 2} = T_2 - T_1} \Rightarrow \text{for all problems.}$$

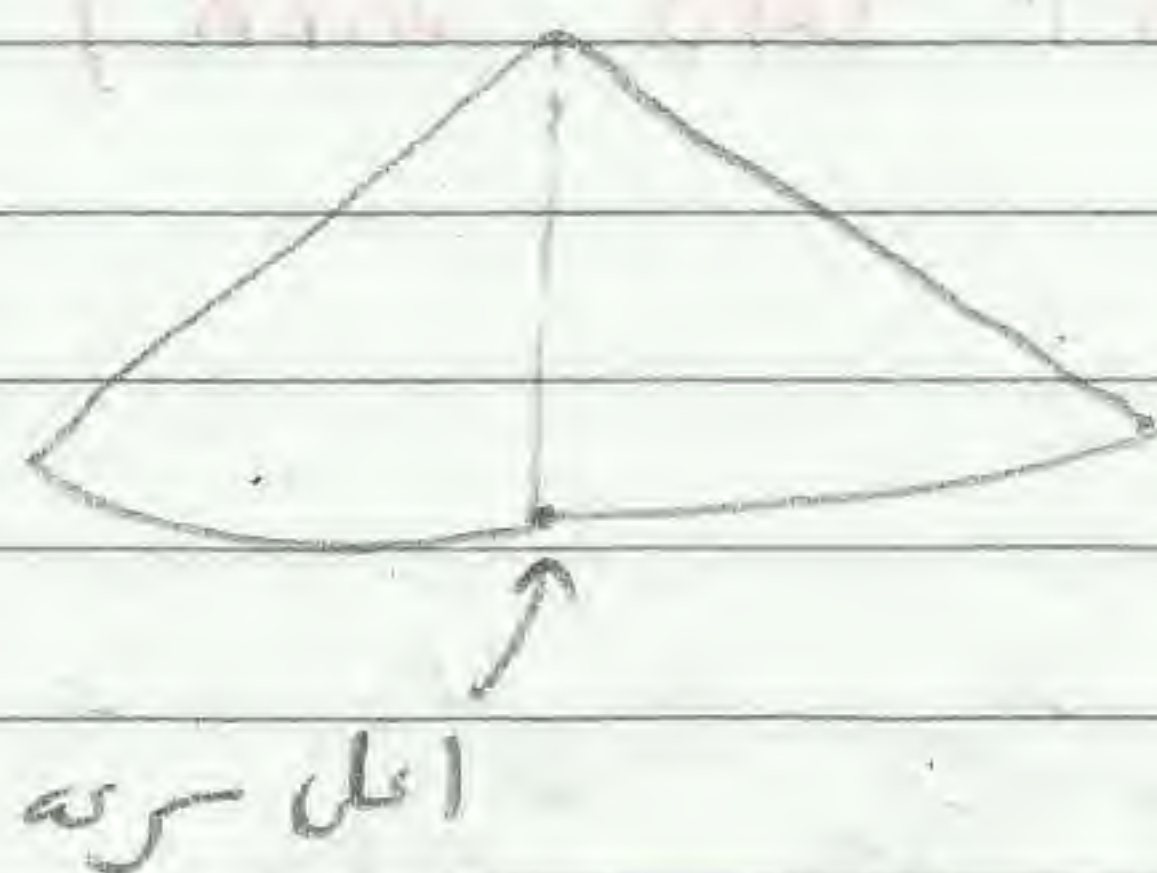
for conservative

بجانبه على ال energy

$$V_1 - V_2 = U_{1 \rightarrow 2} = T_2 - T_1$$

$$\boxed{T_1 + V_1 = T_2 + V_2}$$

كانه بنكه الطاقة



④ Kinetic potential = قدر

المرکز



Kinetic potential = zero







Another solution

$$T_1 + V_1 = T_2 + V_2$$

← لا نحتاج weight أو spring  
في هذه الحالة

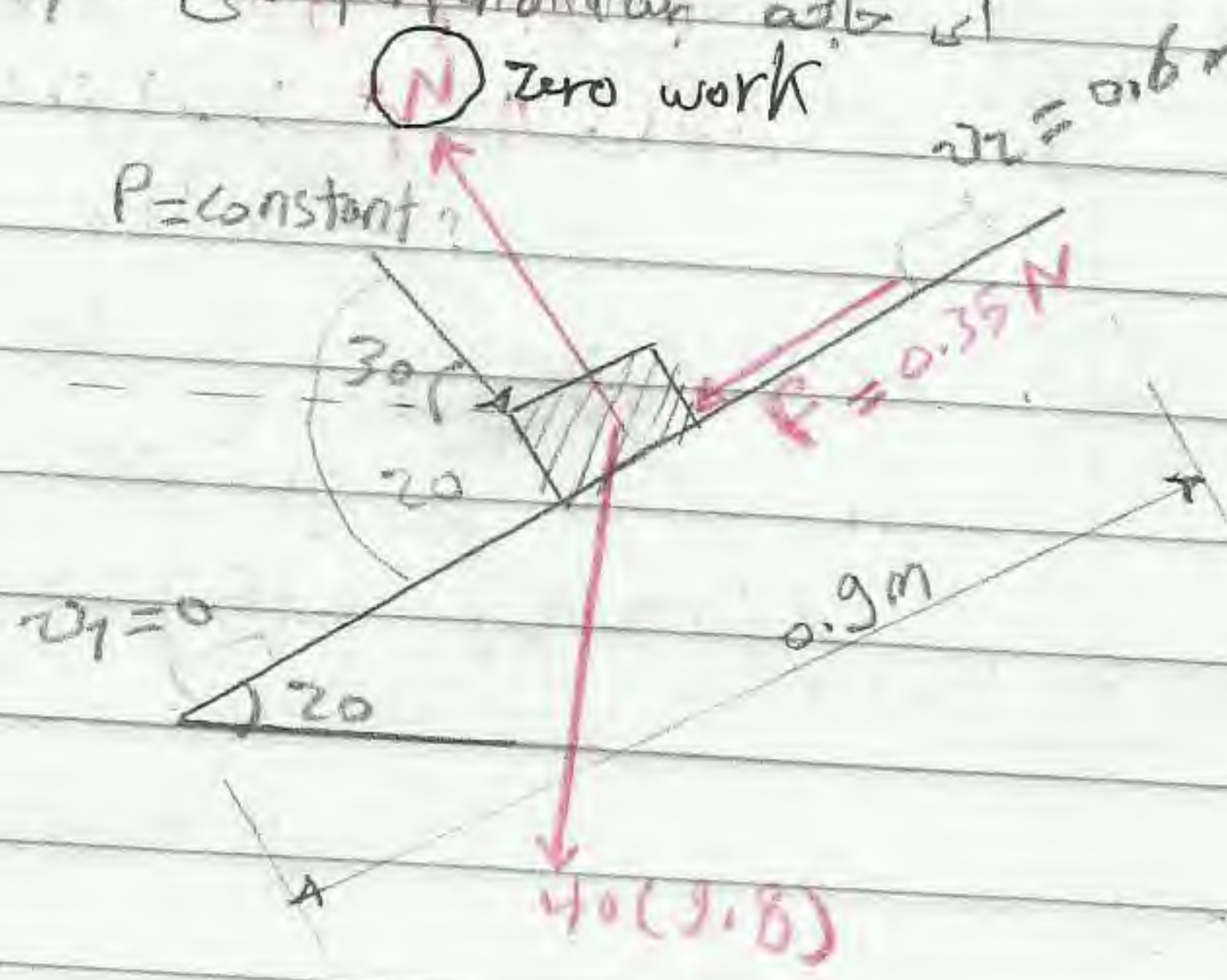
$$V_1 = V_{1g} + V_{1s} = mg(0) + \frac{1}{2} k (0.1)^2$$

$$V_2 = V_{2g} + V_{2s} = -(9)(9.8)(0.15) + \frac{1}{2} k (0.15)^2$$



By using 3 methods.

في 3 طرق مختلفة باستخدام المبدأ الشغل-الطاقة  
 (1) المبدأ الشغل-الطاقة  
 (2) المبدأ الشغل-الطاقة  
 (3) المبدأ الشغل-الطاقة



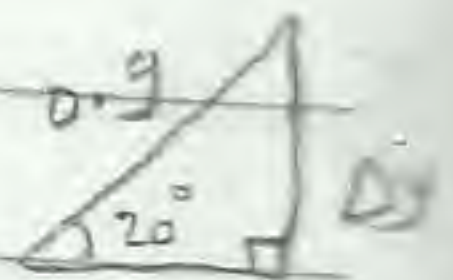
$m = 40 \text{ Kg}$   
 $\mu_k = 0.35$

calculate:  $P$  such that  $v_1 = 0$ ,  $v_2 = 0.6 \text{ m/s}$  in a distance  $0.9 \text{ m}$ .

$$U_{\text{all forces}} = T_2 - T_1$$

$$U_{\text{weight}} + U_{\text{force } P} + U_{\text{friction}} = T_2 - T_1$$

$$U_{\text{weight}} = -mg |\Delta y| = -40(9.8) [0.9 \sin 20]$$



$$U_{\text{force } P} = Fd \cos \theta = P(0.9) \cos 50$$

displacement force at an angle  
 المسافة التي قطعها الجسم في اتجاه القوة

$$U_{\text{friction}} = -F (\text{Path length}) = -(0.35 N) (0.9) = [0.35 (P \sin 50 + 40(9.8) \cos 20)] (0.9)$$

$$N - P \sin(50) - w \cos 20 = m(0)$$



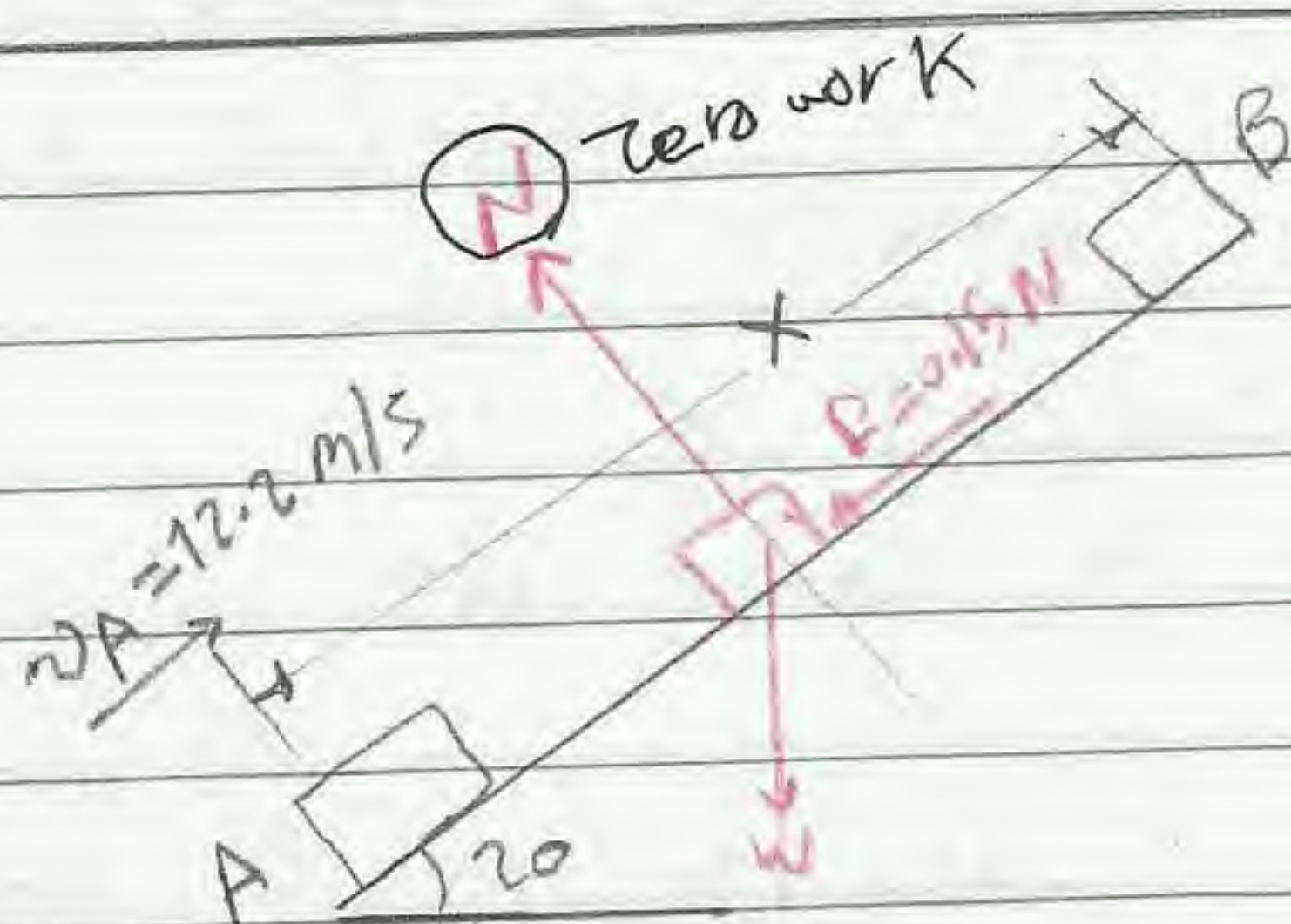
$$T_1 = \frac{1}{2} m v_1^2 = 0$$

$$T_2 = \frac{1}{2} m v_2^2 = \frac{1}{2} (40) (0.6)^2$$

لو فركت في اتجاه اليمين  $\leftarrow$  فركت في اتجاه اليمين  $\leftarrow$  فركت في اتجاه اليمين  $\leftarrow$

لو فركت في اتجاه اليسار  $\leftarrow$  فركت في اتجاه اليسار  $\leftarrow$  فركت في اتجاه اليسار  $\leftarrow$

problem (13.6) page 771



$$m = 22.7 \text{ kg}$$

$$\mu_k = 0.15$$

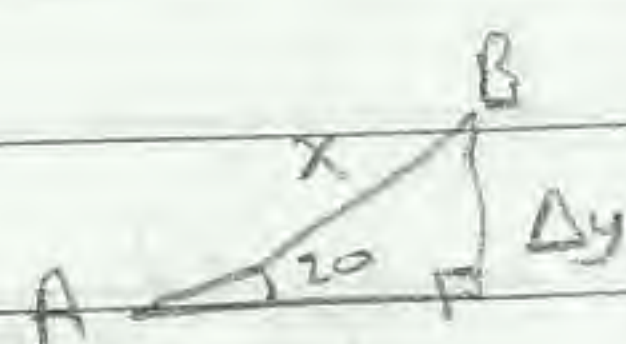
calculate: max travel distance  $x$   $v_B = 0$  (السرعة النهائية صفر)  
,  $(v_A)_{\text{return}}$   $\leftarrow$  the energy dissipated due to friction.

$$N - W \sin 20 = m(0)$$

$$U_{\text{all forces}} = T_2 - T_1$$

$$\Rightarrow U_{\text{weight}} + U_{\text{friction}} = T_B - T_A$$

$$U_{\text{weight}} = -mg |\Delta y| = -(22.7)(9.8) [x \sin 20]$$



$$U_{\text{friction}} = -[0.15 (22.7(9.8) \cos 20^\circ)] x$$



$$T_A = \frac{1}{2} m v_A^2 = \frac{1}{2} (22.7) (12.2)^2$$

$$T_B = \frac{1}{2} m v_B^2 = 0$$

$$x = \checkmark m$$

② motion from B → A

$$U_{B \rightarrow A}^{\text{weight}} + U_{B \rightarrow A} = T_A - T_B$$

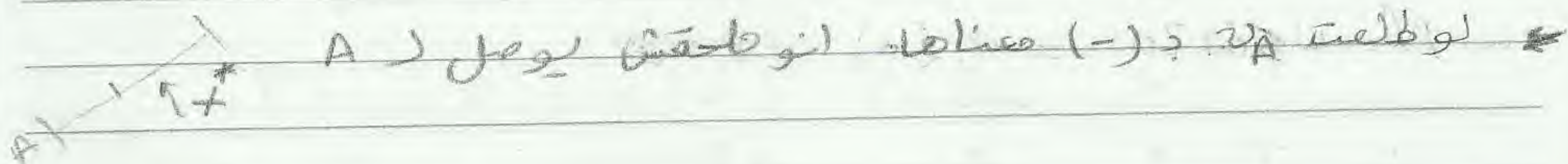
$$U_{B \rightarrow A}^{\text{weight}} = + mg (\Delta y) = + (22.7) (9.8) x \sin 20$$

$$U_{B \rightarrow A}^{\text{friction}} = - 0.15 (22.7 (9.8) \cos 20) x$$

$$T_B = - \frac{1}{2} m v_B^2 = 0$$

$$T_{A(\text{return})} = \frac{1}{2} (22.7) v_{A(\text{return})}^2$$

$$v_A = v_{A(\text{return})} \quad \text{Friction لا يغير السرعة}$$



$$\begin{aligned} \textcircled{3} \quad \Delta T &= (T_A)_{\text{return}} - (T_A)_{\text{initial}} \\ &= \frac{1}{2} (22.7) v_{A(\text{return})}^2 - \frac{1}{2} (22.7) (12.2)^2 \end{aligned}$$

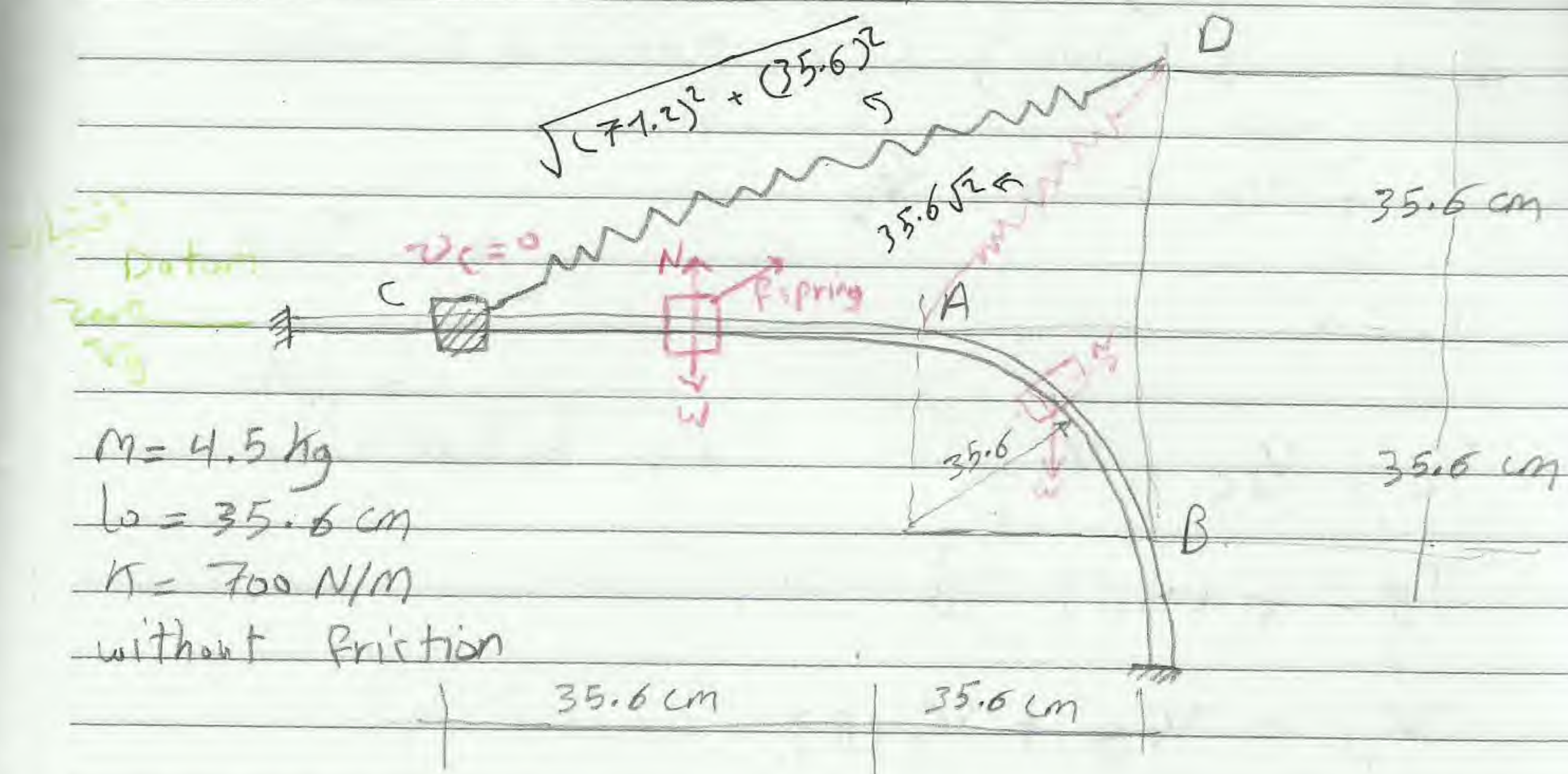
$$U_{A \rightarrow A}^{\text{friction}} = T_{A \text{ return}} - T_{A \text{ up}}$$

$$- 2 u = \Delta t$$

Work done by friction



Problem (13.66) page 795



calculate the speed & Normal reaction at i) A, ii) B

motion from C  $\rightarrow$  A (driving force spring)

$$T_C + V_C = T_A + V_A \quad \text{or} \quad \left( \frac{U}{C \rightarrow A} = T_A - T_C \right)$$

$$T_C = \frac{1}{2} m v_C^2 = 0, \quad T_A = \frac{1}{2} (4.5) v_A^2 = ??$$

$$V_C = \frac{1}{2} k x_C^2 = \frac{1}{2} (700) \left[ \frac{\sqrt{(71.2)^2 + (35.6)^2} - 35.6}{100} \right]^2 \quad \text{CD-b}$$

cm  $\leftarrow$  mio 100  $\rightarrow$

$$V_A = \frac{1}{2} k x_A^2 = \frac{1}{2} (700) \left[ \frac{35.6\sqrt{2} - 35.6}{100} \right]^2 \quad \text{AD-b}$$

$$v_A = v$$

$$\left( \frac{U}{C \rightarrow A} = \frac{1}{2} k x_C^2 - \frac{1}{2} k x_A^2 \right) \quad \text{و حسب القانون الثاني}$$



motion from  $\leftarrow$  B  $\rightarrow$  A  $\rightarrow$  B  $\rightarrow$  A  
 (weight & spring)

$$T_c + V_c = T_B + V_B$$

$$T_c = 0$$

$$V_c = V_{sc} + V_{gc} = \quad \text{As before}$$

$$T_B = \frac{1}{2} (4.5) v_B^2 = ??$$

$$V_B = V_{sB} + V_{g(B)}$$

$$0 = \frac{1}{2} (700) \left[ \frac{71.2 - 35.6}{100} \right]^2 - 4.5 (9.8) (0.356)$$

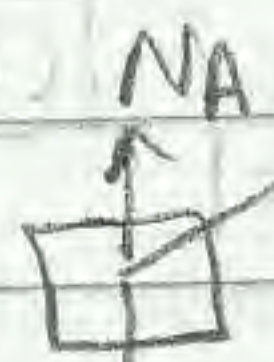


$$v_B = v$$

Normal reaction at A



Be Fore



$$w = mg = 4.5 (9.8)$$

$$F_{spring} = Kx = 700 \left[ \frac{35.6\sqrt{2} - 35.6}{2} \right]$$

$$a_n = 0$$

$$a_t = ?$$

$$4.5 (9.8) - N_A = F_{spring} \sin 45^\circ = m (0)$$

a F ter



$$4.5 (9.8)$$

$$a_n = \frac{v_A^2}{0.356}$$

$$a_t = ?$$

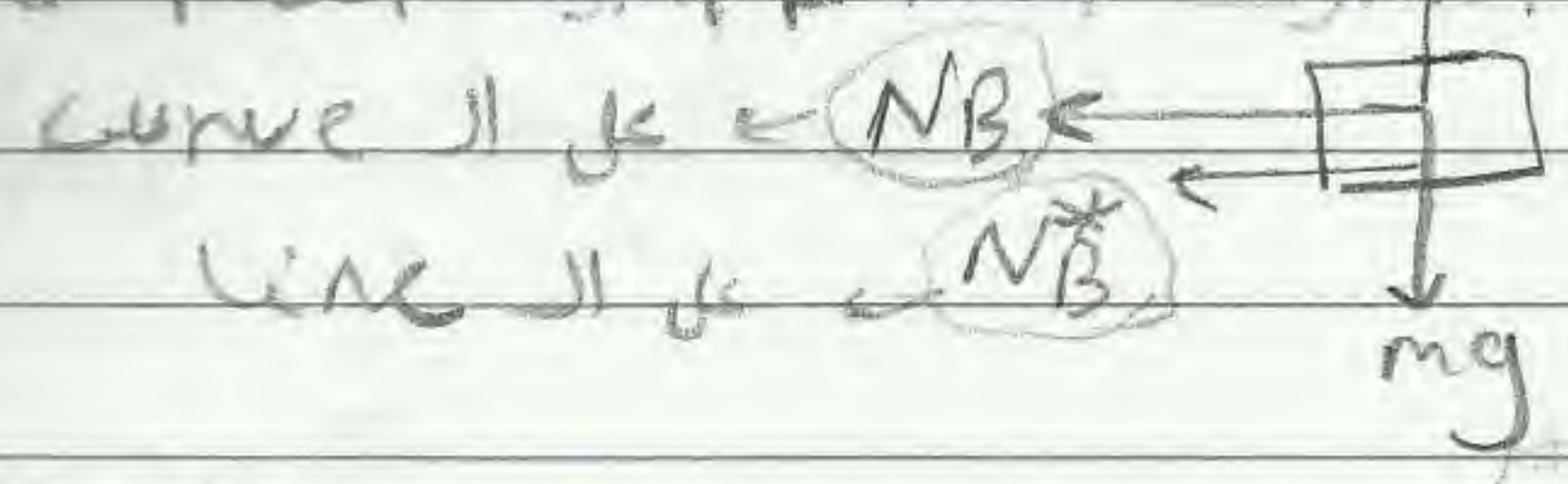
$$(4.5) (9.8) - N_A^* - F_{spring} \sin 45^\circ = 4.5 \left( \frac{v_A^2}{0.356} \right)$$

$$N_A^* = \checkmark$$

لو طلب ال change بيتر



Normal reaction at B



$$N_B^* = 0$$

$$N_B = \frac{F}{2}$$

$$\vec{F} = m \cdot \vec{a} \quad \vec{F} = m \cdot \frac{dv}{dt} \quad \int \vec{F} dt = m \cdot \int dv \quad \vec{F} \cdot \Delta t = m \cdot \Delta v$$

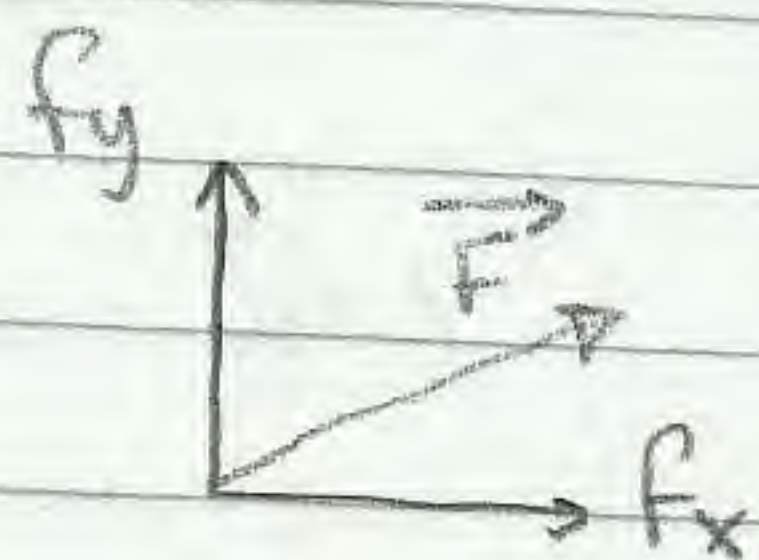
$$\vec{F} = m \cdot \frac{dv}{dt} \quad \vec{F} \cdot dt = m \cdot dv \quad \int \vec{F} dt = m \cdot \int dv$$



# Kinetics of a particle: Impulse & momentum principle (Force-velocity-Time principle)

## 1) Impulse

$$\vec{I} = \int_{t_1}^{t_2} \vec{F} dt \quad \begin{cases} I_x = \int_{t_1}^{t_2} F_x dt \\ I_y = \int_{t_1}^{t_2} F_y dt \end{cases}$$

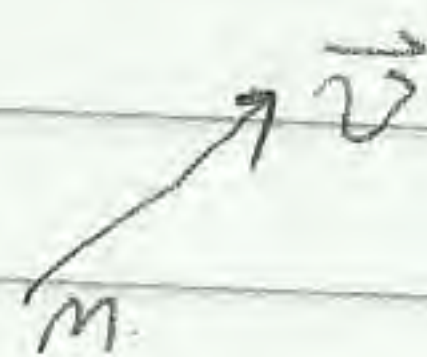


Special case :-  $\vec{F} = \text{constant}$

$$\vec{I} = \vec{F} (\Delta t) \quad \begin{cases} I_x = F_x (\Delta t) \\ I_y = F_y (\Delta t) \end{cases}$$

## 2) Momentum $\vec{p}$ or $\vec{l}$

$$\vec{p} = m \vec{v} \quad \begin{cases} p_x = m v_x \\ p_y = m v_y \end{cases}$$



## 3) Impulse - Momentum relationship.

$$\vec{F} = m \vec{a} \Rightarrow \vec{F} = m \frac{d\vec{v}}{dt} \Rightarrow \int_{t_1}^{t_2} \vec{F} dt = m \int_{\vec{v}_1}^{\vec{v}_2} d\vec{v} \quad \text{mass is constant}$$

$$\int_{t_1}^{t_2} \vec{F} dt = m \vec{v}_2 - m \vec{v}_1 \Rightarrow \boxed{\vec{I} = \Delta \vec{p}}$$

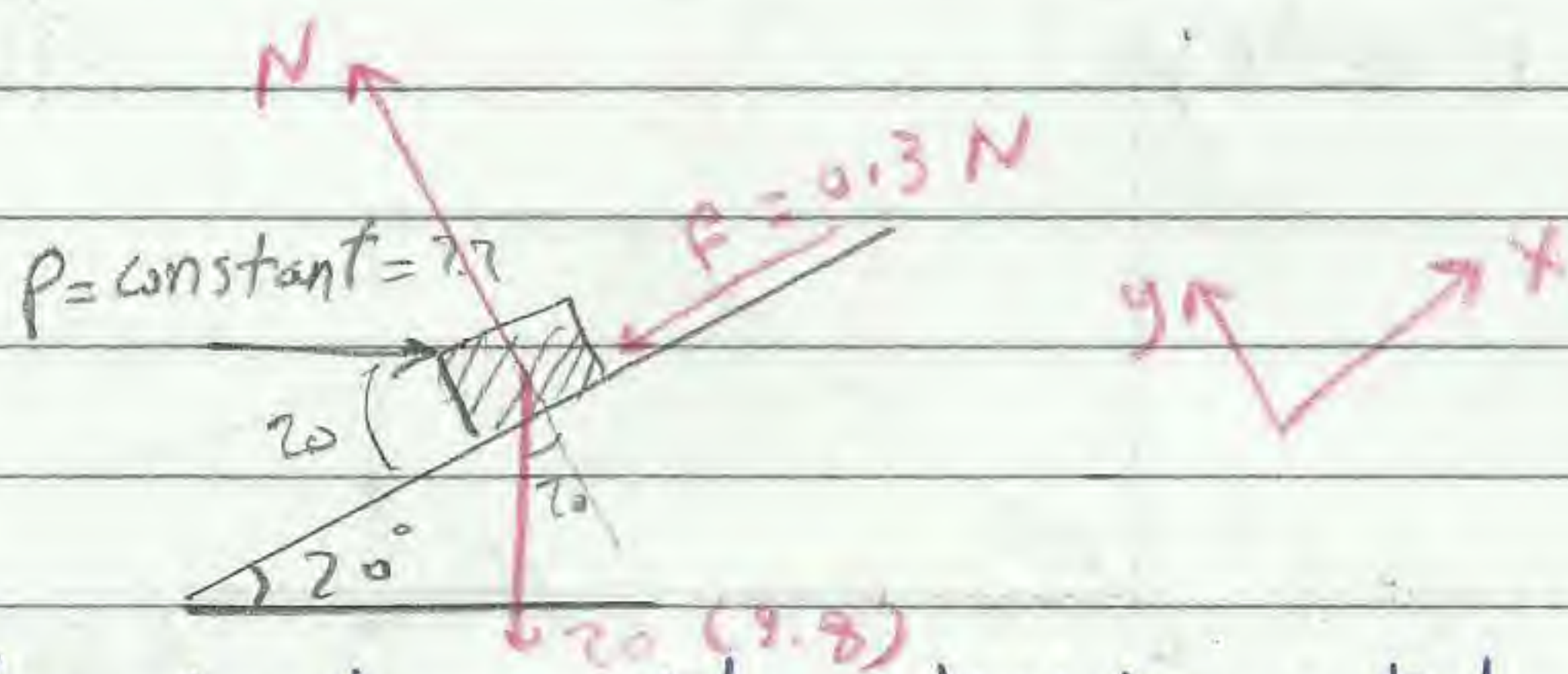
$$\int F_x dt = m (v_2)_x - m (v_1)_x$$

$$\int F_y dt = m (v_2)_y - m (v_1)_y$$



problem (13-126) page 814

By using 3 methods



$$m = 20 \text{ Kg}$$

$$M_k = 0.3$$

$$v_1 = 0$$

$$v_1 = 15 \text{ m/s} \quad (\text{time } 0.5)$$

calculate the horizontal force  $P = ??$

$$I_x = \Delta P_x$$

$$\int_0^6 (P \cos 20^\circ - 70(9.8) \sin 20^\circ - 0.3N) dt = 70(v_2)_x - 70(0)$$

(دوئے طالع و قالش  $t_1 = 0$ )

$$\underbrace{(P \cos 20 - 20(9.8) \sin 20 - 0.3N)}_{\sum F_x} \underbrace{(\Delta t)}_{\Delta t} = 20(15) - 20(0) \quad \text{--- (2)}$$

$$\hat{Y}_y = \Delta P_y$$

$$\int_0^6 (N - 20(9.8) \cos 20 - P \sin 20) dt = 20(v_2)_y - 20(0)$$

$$(N - \gamma_0(y, \theta) \cos \gamma_0 - \rho \sin \gamma_0) (b) = \gamma_0(0) - \gamma_0(0) = 0 \quad (2)$$

Another solution (force-acceleration method)

$$F_x = \text{max} : P \cos 20 - 20(9.8) \sin 20 - 0.3 \cdot N = 20 \text{ a}$$

$$F_y = m a_y : N - 70(9.8) \cos 20 - P \sin 20 = 70(0)$$

$$v_2 = v_1 + at$$

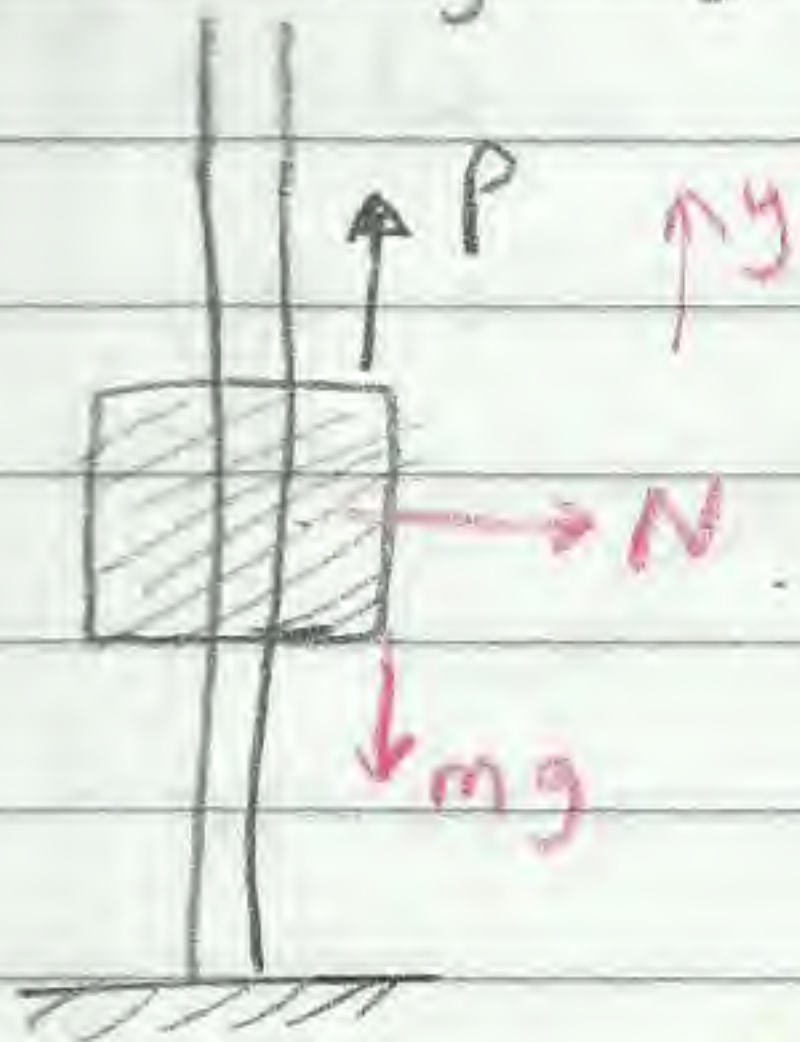
$$15 = 0 + a(0) \Rightarrow$$

$$a = 2.5$$



problem (13-136) page 815

$m = 2 \text{ kg}$  starts from rest, calculate  $v_{2s}$ ,  $v_{3s}$

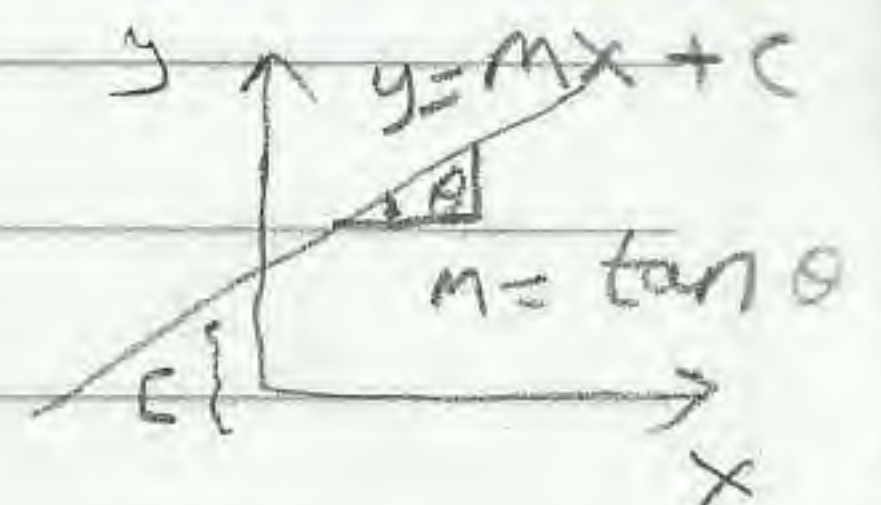


Frictionless

$P$  (Newtons)

40

$$P = 20t$$



الزمن الذي يتحرك فيه

$$A = \frac{1}{2} (2) (40)$$

$t$  (s)

$$\int_x = \Delta P_x$$

$$\int N dt = m(v_2)_x - m(v_1)_x$$

$$N(\Delta t) = 2(0) - 2(0)$$

$$N = 0$$

$$\int_y = \Delta P_y$$

at  $t = 2s$

$$\int_0^2 (P - 2(9.8)) dt = m(v_2)_y - m(v_0)_y$$

constant  $g$  and  $P \rightarrow$

$$\left[ \int_0^2 20t dt \right] - 2(9.8)(2) = 2 v_{2s}$$

$$\left[ 10t^2 \right]_0^2 - 2(9.8)(2) = 2 v_{2s}$$

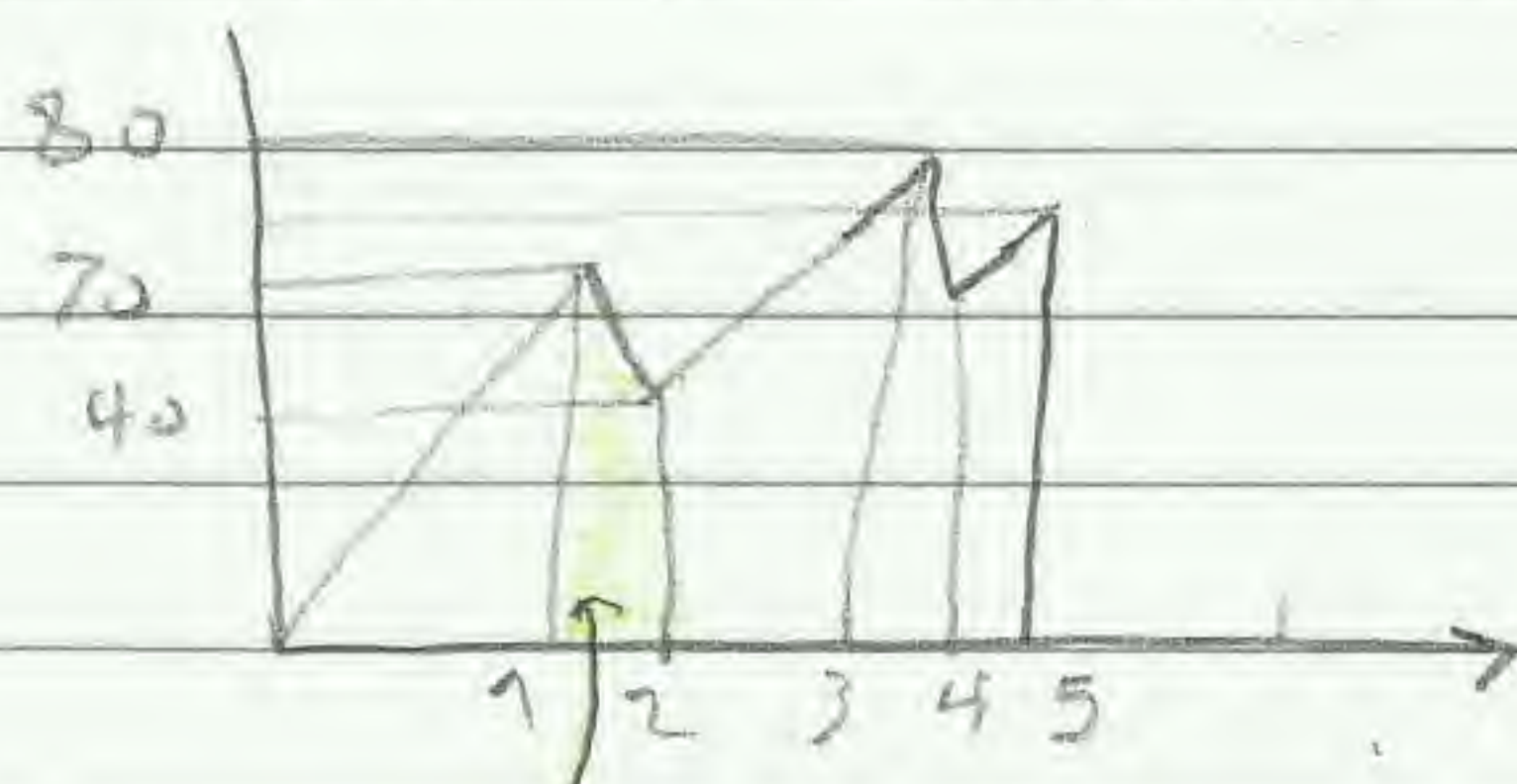
$$v_{2s} = \frac{40 - 2(9.8)(2)}{2}$$

$$\text{at } t = 3s \quad \int_0^3 [P - 2(9.8)] dt = m v_{3s} - m v_0$$

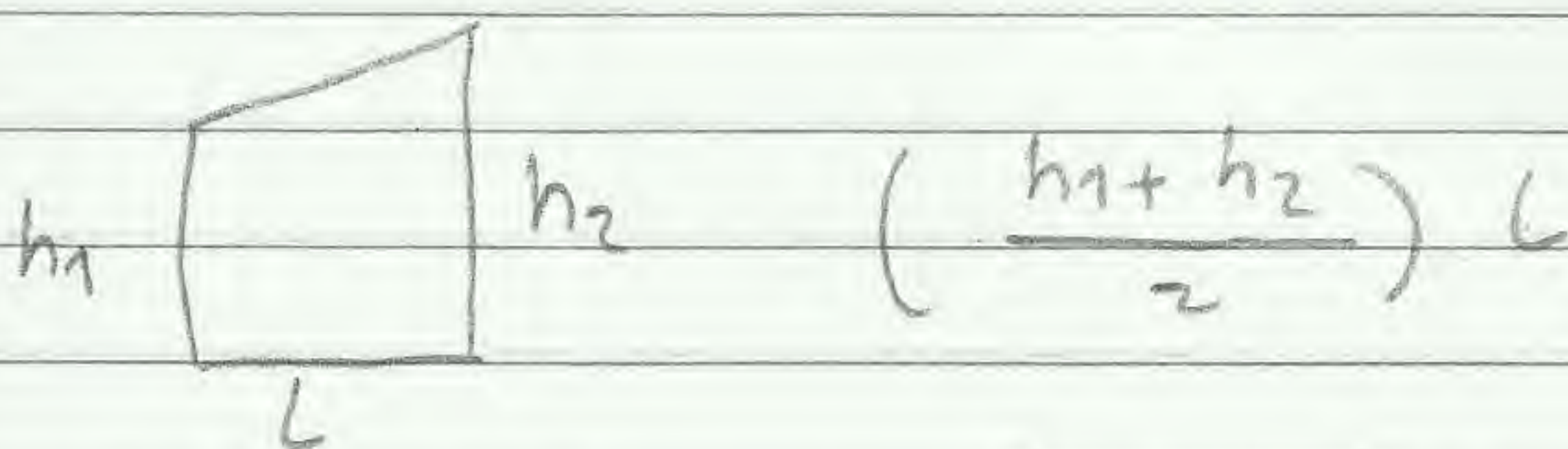
$$\int_0^2 20t dt + \int_2^3 40 dt - 2(9.8)(3) = 2 v_{3s} - 2(0)$$

$$40 + 40(1) - 2(9.8)(3) = 2 v_{3s}$$





$$\frac{40 + 70}{2} \times (2 - 1)$$

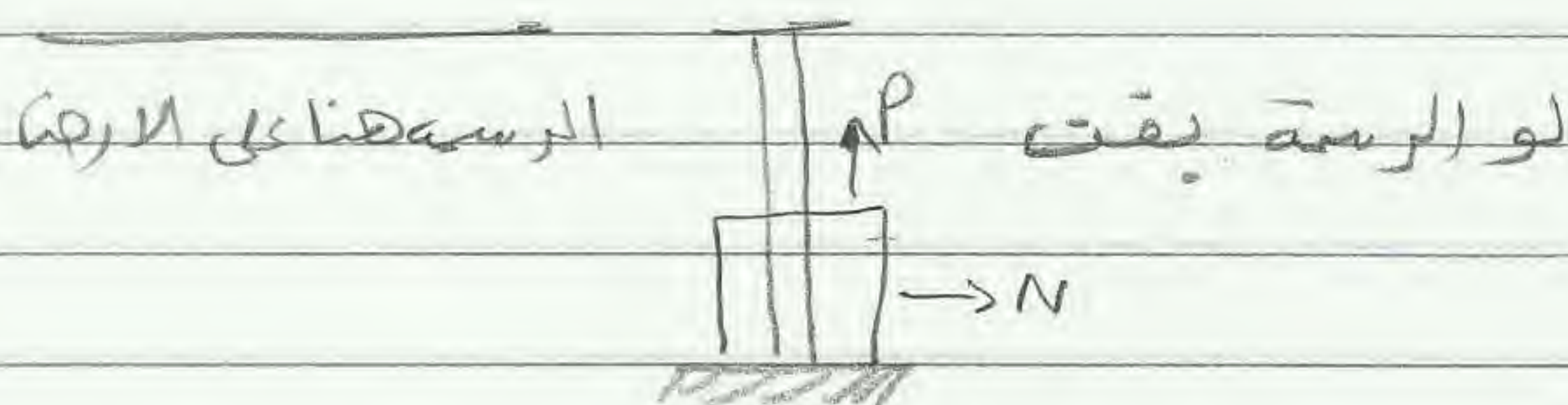


في كل جانب مثل الوقت الى يكون عند الجسم لاكنه لحظيا

$$\int_0^2 20t \, dt + \int_2^3 40 \, dt - (2)(9.8)(t) = 0$$

$0 = v_f$  و  $v_0 = 0$

start from rest & come to rest



فيكون في N + الحركة عند حدي من عند لا نرى الاول يكون وزن الجسم

أكبر من القوة P و بعد أعصاب الحركة عند الوقت الذي يصبح فيه P = 0

وبالتالي تكون فترة التكامل لا تبدأ من الصفر  
 $\int_x$  ← المطلوب الحساب عند  
 وقت بداية الحركة

$$\boxed{P = 20t} \Rightarrow t = \frac{\frac{2(9.8)}{20}}{20}$$

مثال



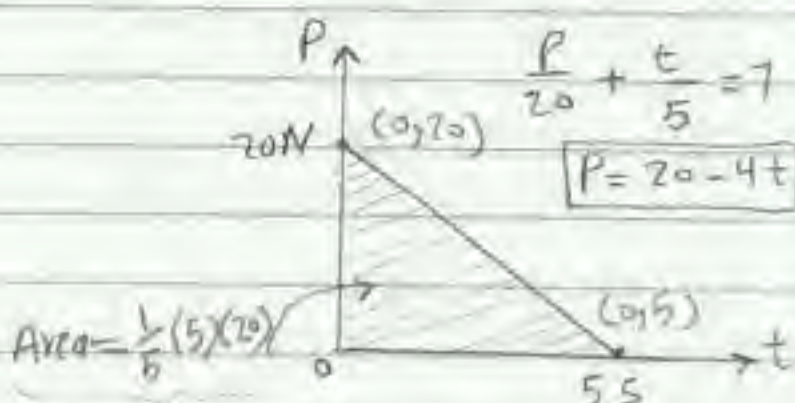
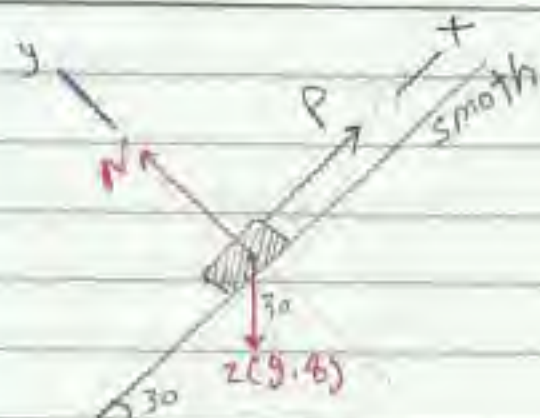
## Impulse &amp; momentum principle

$$\boxed{J_x = \Delta P_x} \Rightarrow \int_{t_1}^{t_2} F_x dt = m(v_2)_x - m(v_1)_x$$

$$\boxed{J_y = \Delta P_y} \Rightarrow \int_{t_1}^{t_2} F_y dt = m(v_2)_y - m(v_1)_y$$

problem (13.138) page 816

mass = 2 kg

 $v_0 = 0$ Calculate:  $t_{55}$ , time when the velocity  $v = 0$ friction is negligible  $\mu = 0$  N constant. \*

$$\boxed{J_x = \Delta P_x} \quad \int_0^5 (P - 12)(9.8) \sin 30^\circ dt = 2v_{55} - 2(0)$$

$$\int_0^5 P dt - [2(9.8) \sin 30^\circ](5) = 2v_{55}$$

$$v_{55} = \checkmark$$

when the block come to stop  $v_{\text{final}} = 0$   
 at time  $t = ??$

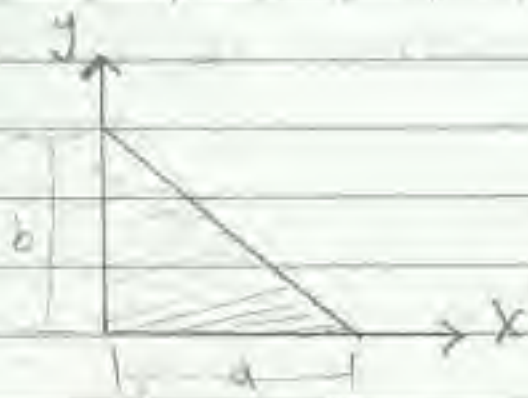
$$\int_0^t P - (2(9.8) \sin 30^\circ)t = 2(v_{\text{final}}) - 2(v_0)$$

$= 50$   
 $P = 20$  at  $t = 5$  s variables

$$t = \checkmark$$



Note:

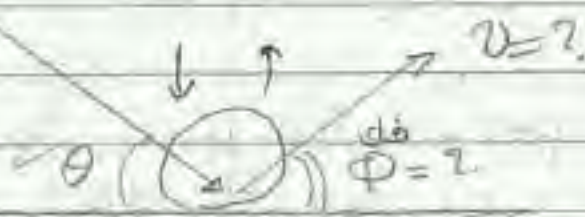


$$\frac{y}{b} + \frac{x}{a} = 1$$

$$\int y \, dx = \text{Area under curve}$$

Impact with a fixed surface:

u (initial) to go  $u = v$



Given:  $u, \theta, \Delta t$

Required:  $v, \phi$

$$\int_{t_x} = \Delta p_x$$

$$\int_0^{\Delta t} (F_x) \, dt = m(v \cos \phi) - m(u \cos \theta)$$

Force impulse  $\rightarrow$   
 $x = \text{normal}$

$$m v \cos \phi = m u \cos \theta \quad \text{--- (1)}$$

Tangential component of velocity is constant

$$\int_{t_y} = \Delta p_y$$

$$\int_0^{\Delta t} (N - mg) \, dt = m(v \sin \phi) - m(u \sin \theta)$$

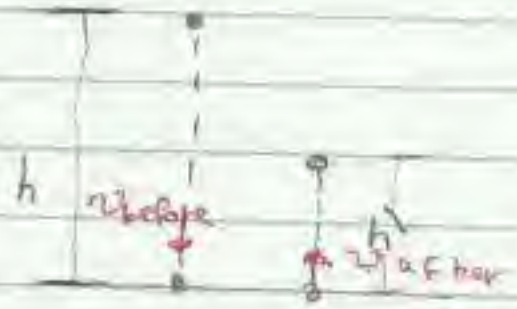
--- (2)



coefficient of restitution ( $e$ )

$$v_{\text{after}} = e v_{\text{before}}$$

تسوية على نوع المادة  
(مطابق المرونة)



$$e \in [0, 1]$$

$$v \sin \phi = e (u \sin \theta) \quad \text{--- (2)}$$

The velocity perpendicular to the surface is reversed & multiplied by  $e$ .

Example (13.14) page 828

$$e = 0.9$$



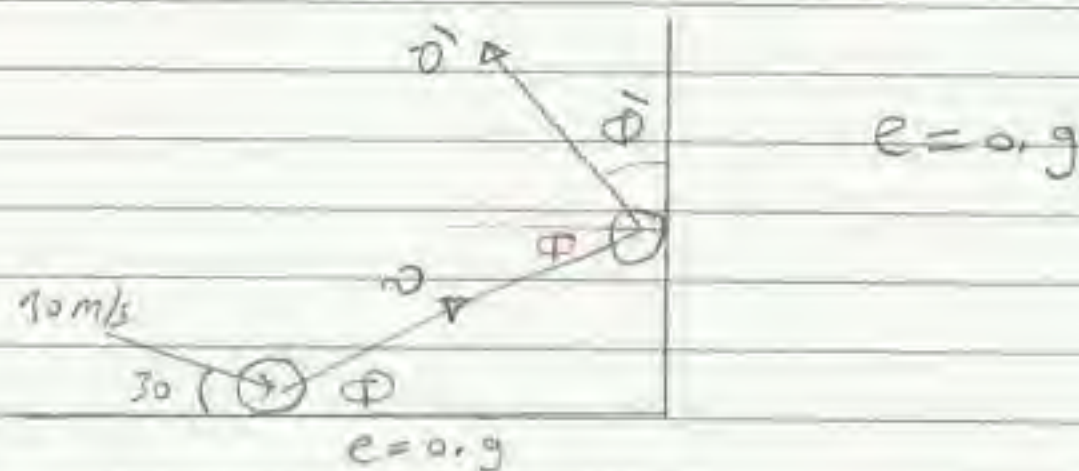
$$v \cos \phi = 10 \cos 30 \quad \text{--- (1)}$$

$$v \sin \phi = 0.9 (10 \sin 30) \quad \text{--- (2)}$$

$$\frac{(2)}{(1)} \quad \tan \phi = 0.9 \tan 30$$

$$v = \checkmark$$





Impact 1

$$\left. \begin{aligned} v \cos \phi &= 10 \cos 30 \\ v \sin \phi &= 0.9 (10 \sin 30) \end{aligned} \right\} \begin{aligned} v \\ \phi \end{aligned}$$

Impact 2

$$\left. \begin{aligned} v' \cos \phi' &= v \sin \phi \\ v' \sin \phi' &= 0.9 (v \sin \phi) \end{aligned} \right\} \begin{aligned} v' \\ \phi' \end{aligned}$$

